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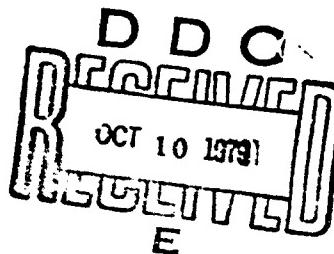
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ROTOR-BEARING DYNAMICS TECHNOLOGY DESIGN GUIDE
Part III
Tapered Roller Bearings

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February 1979

Interim Report for Period April 1977 - December 1978

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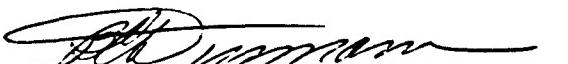


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FOREWORD

This report was prepared by Shaker Research Corporation under USAF Contract No. AF33615-76-C-2038. The contract was initiated under Project 3048, "Fuels, Lubrication, and Fire Protection," Task 304806, "Aerospace Lubrication," Work Unit 30480685, "Rotor-Bearing Dynamics Design."

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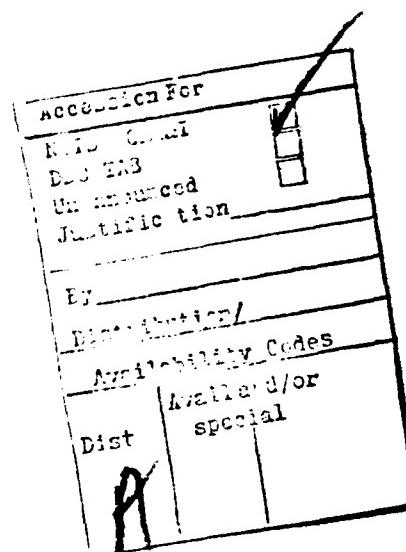


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NOMENCLATURE

<u>Symbol</u>	<u>Description</u>	<u>Units</u>
b_x	Semi-width of contact ellipse at x	in.
B_1	Corner break at roller small end	in.
B_2	Corner break at roller big end	in.
B_{ij}	Damping component, change of force in i direction due to velocity in j direction; i = x, y, z; j = x, y, z.	<u>lb-sec</u> in
\underline{B}_N	Damping matrix $\begin{bmatrix} (\underline{B}_N)_{\text{lineal}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & (\underline{B}_N)_{\text{angular}} \end{bmatrix}$	<u>lb-sec</u> in
$(\underline{B})_{\text{lineal}}$	Damping matrix due to lateral velocities $\begin{bmatrix} B_{xx} & B_{xy} \\ B_{yx} & B_{yy} \end{bmatrix}_{\text{lineal}}$	<u>lb-sec</u> in
$(\underline{B})_{\text{angular}}$	Damping matrix due to angular velocities $\begin{bmatrix} B_{xx} & B_{xy} \\ B_{yx} & B_{yy} \end{bmatrix}_{\text{angular}}$	<u>in-lb-sec</u> radian
C_i	A constant, $C_i = \begin{cases} 1 & \text{for } i = 1 \\ -1 & \text{for } i = 2 \end{cases}$	-
d	Roller diameter at midpoint of effective length of roller	in.
d_x	Roller diameter at x	in.
E	Pitch diameter at midpoint of effective length	in.
E_E	Modulus of elasticity for roller	lbs/in^2

E_R	Modulus of elasticity for race body	lbs/in^2
E_x	Pitch diameter at x	in.
F_c	Roller centrifugal force	lbs.
F_i	External applied force, $i = x, y, z$	lbs.
F'_i	Reaction force, positive in direction opposite to displacements, $i = x, y, z$	lbs.
F	Force Matrix = $\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}$	lbs.
G	Distance along roller cone element from extreme end of effective length to point where crown drop is measured	in.
H	Roller crown radius minus the rise of the arc at midpoint of effective length	in.
I_{cg}	Moment of inertia about roller center of gravity	lbs-in^2
K	Roller-race stiffness	lbs/in.
K_{ij}	Stiffness component, change of force in i direction due to displacement in j direction. $i = x, y, z; j = x, y, z$	lbs/in.
K_N	Stiffness matrix $\begin{bmatrix} (K_N)_{\text{lineal}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & (K_N)_{\text{angular}} \end{bmatrix}$	
$(K)_N$ lineal	Stiffness matrix due to lateral displacements	lbs/in.
	$\begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix}_{\text{lineal}}$	
$(K)_N$ angular	Stiffness matrix due to angular rotations	$\frac{\text{in-lb}}{\text{rad}}$
	$\begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix}_{\text{angular}}$	

l	Perpendicular distance from the line of action of flange reaction, P_3 , to roller centerline at midpoint of effective length	in.
l_e	Effective length of roller load carrying surface	in.
l_F	Length of flat portion of roller measured along roller cone element	in.
l_T	Total length of roller measured parallel to roller axis between sharp intersections of end faces with roller cone elements	in.
m	Mass of roller	lbs.
M_G	Roller gyroscopic moment	lbs-in.
M_i	External applied moment, $i = x, y, z$	lbs-in.
M'_i	Reaction moment, $i = x, y, z$	lbs-in.
M_1	Outer race/roller contact moment	lbs-in.
M_2	Inner race/roller contact moment	lbs-in.
n	Number of rollers	
N_1	Outer ring rotational speed	rad/sec.
N_2	Inner ring rotational speed	rad/sec.
p_x	Contact unit loading	lbs/in.
p'_x	Current estimate of contact unit loading	lbs/in.
p_D	Diametral clearance	in.
P_{1q}	Outer contact load on qth roller	lbs.
P_{2q}	Inner contact load on qth roller	lbs.
P_{3q}	Flange reaction on qth roller	lbs.
q	Roller position index	-
R_c	Roller crown radius	in.
R_E	Roller big-end spherical radius	in.
V	Radial distance from roller center line to flange reaction	in.

\underline{w}_N	Column Matrix =	$\begin{bmatrix} \delta_x \\ \delta_y \\ \theta_x \\ \theta_y \end{bmatrix}$
x, y, z	Bearing coordinate system	in.
x_o	Static component of displacement	in.
x'	Dynamic component of displacement	in.
x_A	Big-end extremity of contact pattern measured parallel to roll axis from the midpoint of the effective length	
x_B	Small and extremity of contact pattern measured parallel to roller axis from the midpoint of the effective length	in.
x_A^*	Maximum permissible distance of big-end pattern extremity from midpoint of effective length measured along race	in.
x_B^*	Maximum permissible distance of small-end pattern extremity from midpoint of effective length measured along race	in.
\underline{z}_N	Impedance matrix = $K_N + i v B_N$	

Other notations as defined in text.

GREEK SYMBOLS

α	Angle between roller axis and line of action of flange reaction	radians
β	Outer ring contact angle	radians
γ_1	$d_x \cos\beta/E_x$	-
γ_2	$d_x \cos(\beta-\tau)/E_x$	-
δ	Displacement	in.
δ_x	Linear displacement in x direction	in.
δ_y	Linear displacement in y direction	in.
δ_z	Linear displacement in z direction	in.
Δ	Approach of inner race to outer race at midpoint of effective length	in.
Δ_x	Approach of roller to race at x	in.
Δ_{1q}	Approach of roller to outer race (cup) at qth roller	in.
Δ_{2q}	Approach of roller to inner race (cone) at qth roller	in.
ϵ_i	Residues of simultaneous equations	-
η_E	Roller elastic constant = $\frac{4(1 - v_E^2)}{E_E}$	$\frac{\text{in}^2}{\text{lb.}}$
η_R	Race elastic constant = $\frac{4(1 - v_R^2)}{E_R}$	
θ_x	Angular rotation about x axis	radians, °
θ_y	Angular rotation about y axis	radians, °
θ_z	Angular rotation about z axis	radians, °
ν	Frequency of rotation	rad/sec.
v_E	Poisson's ratio for roller	

ν_R	Poisson's ratio for race	
	Material density	lbs/in ³
τ	Included roller cone angle	radians, ^o
ϕ	Circumferential roller position	radians, ^o
ω_R	Angular velocity of roller about its own center	rad/sec.
Ω	Orbital velocity of roller	rad/sec.
∇	Crown drop	in.

SUBSCRIPTS

<u>Symbol</u>	<u>Description</u>
b	Refers to bearing
cg	Refers to center of gravity
E	Refers to roller
F	Refers to flat
g	Refers to gyroscopic
i	Index, $i = 1, 2, 3$ or $i = x, y, z$
i,j	Refers to index of stiffness matrix; i.e., force in i direction due to displacement in j direction
p	Refers to pedestal
q	Refers to roller circumferential position
R	Refers to roller
T	Refers to total
x	Refers to x direction
y	Refers to y direction
z	Refers to z direction
1	Refers to outer race
2	Refers to inner race

SECTION I

INTRODUCTION

The original Rotor-Bearing Dynamics Design Technology Series AFAPL-TR-65-45 (Parts I through X) included a volume, Part IV(1), which presented design data for typical deep-groove and angular contact ball bearings. The data was presented in graphical form and consisted of direct radial stiffness, load carrying capacity, and load levels. In addition design guidelines and limitations were discussed. The major deficiencies of this original volume were that centrifugal effects due to high speed were ignored, and axial and angular stiffness information were omitted.

Subsequent to the publication of Part IV, several extensive treatments of rolling element bearings including elastohydrodynamic, thermal, and cage effects have been published. The computer program of Mauriello, LaGasse, and Jones (2) considers both elastohydrodynamic and cage effects for ball bearings. The more recent computer based design guide prepared by Crecelius and Pirvics (3) treats elastohydrodynamic, thermal, and cage effects for a system of ball and roller bearings.

Thus, very sophisticated analytical tools are available for the design and application of rolling element bearings. Neither of these tools, however, provide the user with the stiffness matrix required for solution of rotor dynamics problems. In addition both computer programs are very large and require an extensive computer facility for use.

Part II(4) of the revised series provided an update of the original Part IV(1). Those aspects of the original Part IV(1) which treated general design aspects of ball bearings, load capacity, speed limitations, etc. were deleted since their coverage is superficial compared to the more sophisticated computer tools now available (2,3). Only those parts directly connected with preparation of input for the rotordynamic response

programs (Part I(5) of the revised series) were retained. The stiffness data included in the original Part IV were also updated.

The present volume (Part III of the revised series) extends the treatment of rolling element bearings to the tapered roller bearing. The complete stiffness matrix is calculated including centrifugal effects. Considerations such as elastohydrodynamic and cage effects are not included since they have little influence on the calculation of tapered roller bearing stiffness. The resulting program (Appendix) is reasonably small and easy to use.

SECTION II

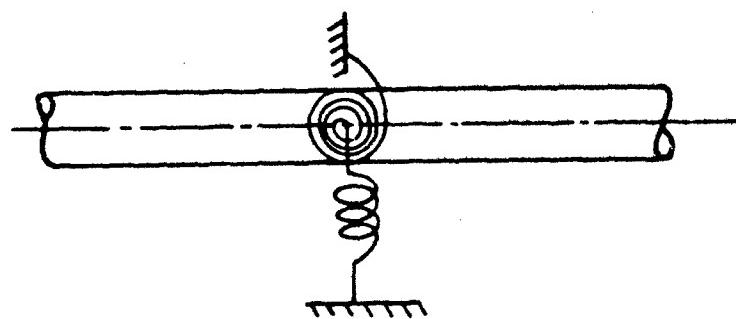
ANALYSIS

2.1 General Bearing Model and Coordinate System

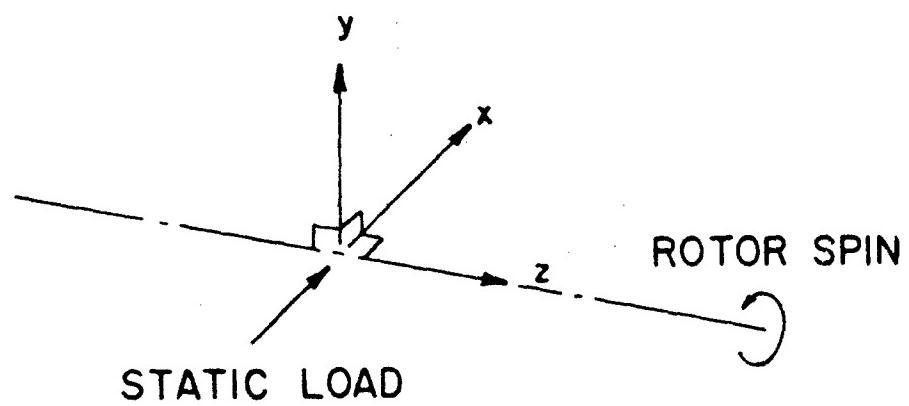
Accurate calculation of the lateral dynamic response of a high-speed rotor depends on realistic characterization of the support bearings. In the most general case, both linear and angular motions are restrained by the support bearings at the attachment location. In the analytical model, the reaction force and the reaction moment of each bearing are felt by the rotor through a single station of the rotor axis. As schematically illustrated in Figure 1a, a coil spring restraining the lateral displacement and a torsion spring which tends to oppose an inclination are attached to the same point of the rotor axis. A complete description of the characteristics of the support bearings, however, involves much more than the specification of the two spring constants. This is because:

- . The lateral motion of the rotor axis is concerned with two displacement components and two inclination components.
- . The restraining characteristics may include cross coupling among various displacement/inclination coordinates.
- . The restraining force/moment may not be temporally in phase with the displacement/inclination.
- . The restraining characteristics of the bearing may be dependent on either the rotor speed or the frequency of vibration, or both.
- . Bearing pedestal compliance may not be negligible.

To accommodate the above considerations, the support bearing characteristics are described in Reference 5 by a four-degrees-of-freedom impedance matrix as defined in Equation (1):



(FIG 1a) Bearing Stiffness Model



(FIG 1b) Bearing Location Coordinate System

$$\underline{R}_N = - \underline{Z}_N \cdot \underline{W}_N \quad (1)$$

where \underline{W}_N is a column vector containing elements which are the two lateral displacements (δ_x , δ_y) and the two lateral inclinations (θ_x , θ_y) of the rotor axis at the bearing station N.

Employing a right-handed Cartesian representation in a lateral plane as depicted in Figure 1b, the z-axis is coincident with the spin vector of the rotor. The x-axis is oriented in the direction of the external static load, and the y-axis is perpendicular to both z and x axes forming the right-handed triad (x, y, z). (δ_x , δ_y) are respectively lateral lineal displacement components of the rotor axis along the (x, y) directions. (θ_x , θ_y) are lateral inclination components respectively in the (z-x, z-y) planes. Note that θ_y is a rotation about the y-axis, while θ_x is a rotation about the negative x-axis.

\underline{Z}_N is a complex (4 x 4 matrix), and in accordance with the common notation for stiffness and damping coefficients, may be expressed as

$$\underline{Z}_N = \underline{K}_N + i v \underline{B}_N \quad (2)$$

where \underline{K}_N is the stiffness matrix and \underline{B}_N is the damping matrix. v is the frequency of vibration. Most commonly, lateral lineal and angular displacements do not interact with each other so that the non-vanishing portions of \underline{K}_N and \underline{B}_N are separate 2 x 2 matrices. That is

$$\underline{K}_N = \begin{bmatrix} (\underline{K}_N)_{\text{lineal}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & (\underline{K}_N)_{\text{angular}} \end{bmatrix} \quad (3)$$

$$\underline{\underline{B}}_N = \begin{bmatrix} (\underline{\underline{B}}_N)_{\text{lineal}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & (\underline{\underline{B}}_N)_{\text{angular}} \end{bmatrix} \quad (4)$$

Accordingly, a total characterization of a support bearing would include sixteen coefficients which make up the 4 (2 x 2) matrices:

$$(\underline{\underline{K}})_{\text{lineal}} = \begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix}_{\text{lineal}} \quad (5)$$

$$(\underline{\underline{B}})_{\text{lineal}} = \begin{bmatrix} B_{xx} & B_{xy} \\ B_{yx} & B_{yy} \end{bmatrix}_{\text{lineal}} \quad (6)$$

$$(\underline{\underline{K}})_{\text{angular}} = \begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix}_{\text{angular}} \quad (7)$$

$$(\underline{\underline{B}})_{\text{angular}} = \begin{bmatrix} B_{xx} & B_{xy} \\ B_{yx} & B_{yy} \end{bmatrix}_{\text{angular}} \quad (8)$$

In the event that the pedestal compliance is significant, then the effective support impedance can be calculated from

$$\underline{\underline{z}}_N = (\underline{\underline{z}}_b^{-1} + \underline{\underline{z}}_p^{-1}) \quad (9)$$

where subscripts "p" and "b" refer to the pedestal and bearing respectively. Note that both pedestal inertia and damping may be included in $\underline{\underline{z}}_p$.

2.2 General Bearing Support Characteristics

The function of a bearing is to restrict the rotor axis to a nominal axis under realistic static and dynamic load environments. Deviation of any particular point of the rotor axis from the nominal line can be characterized by three lineal and two angular displacements. These may be designated as $(\delta_x, \delta_y, \delta_z, \theta_x, \theta_y)$ in accordance with a right-handed Cartesian reference system. The z-coordinate is coincident with the reference axis and is directed toward the spin vector. (θ_x, θ_y) are rotor axis inclinations respectively in the z-x and z-y planes. The x-coordinate is directed toward the predominant static load; e.g., earth gravity. Ideally, the bearing would resist the occurrence of any displacement so that the reaction force system imparted by the bearing to the rotor is generally expressed in matrix notation as

$$\underline{F} = \underline{Z} \cdot \underline{x} \quad (10)$$

\underline{F} is a column vector comprising the five reaction components $(F_x, F_y, F_z, M_x, M_y)$, while \underline{x} is the displacement vector $(\delta_x, \delta_y, \delta_z, \theta_x, \theta_y)$. \underline{Z} is a (5×5) matrix containing the elements Z_{ij} with both indices (i, j) ranging from 1 to 5. The values of Z_{ij} characterize how rotor displacements are being resisted by the bearing.

From the standpoint of dynamic perturbation, distinction is made between a static equilibrium component and a dynamic perturbation component for both the displacements and the reactions. Thus,

$$\underline{x} = \underline{x}_0 + \underline{x}'; \quad \underline{F} = \underline{F}_0 + \underline{F}' \quad (11)$$

$(\underline{x}', \underline{F}')$ are respectively presumed to be infinitesimal in comparison with $(\underline{x}_0, \underline{F}_0)$. Accordingly, Z_{ij} are regarded as dependent on \underline{x}_0 but not on \underline{x}' . To illustrate the idea of perturbation linearization, one may examine the one-dimensional load-displacement curve shown in Figure 2.

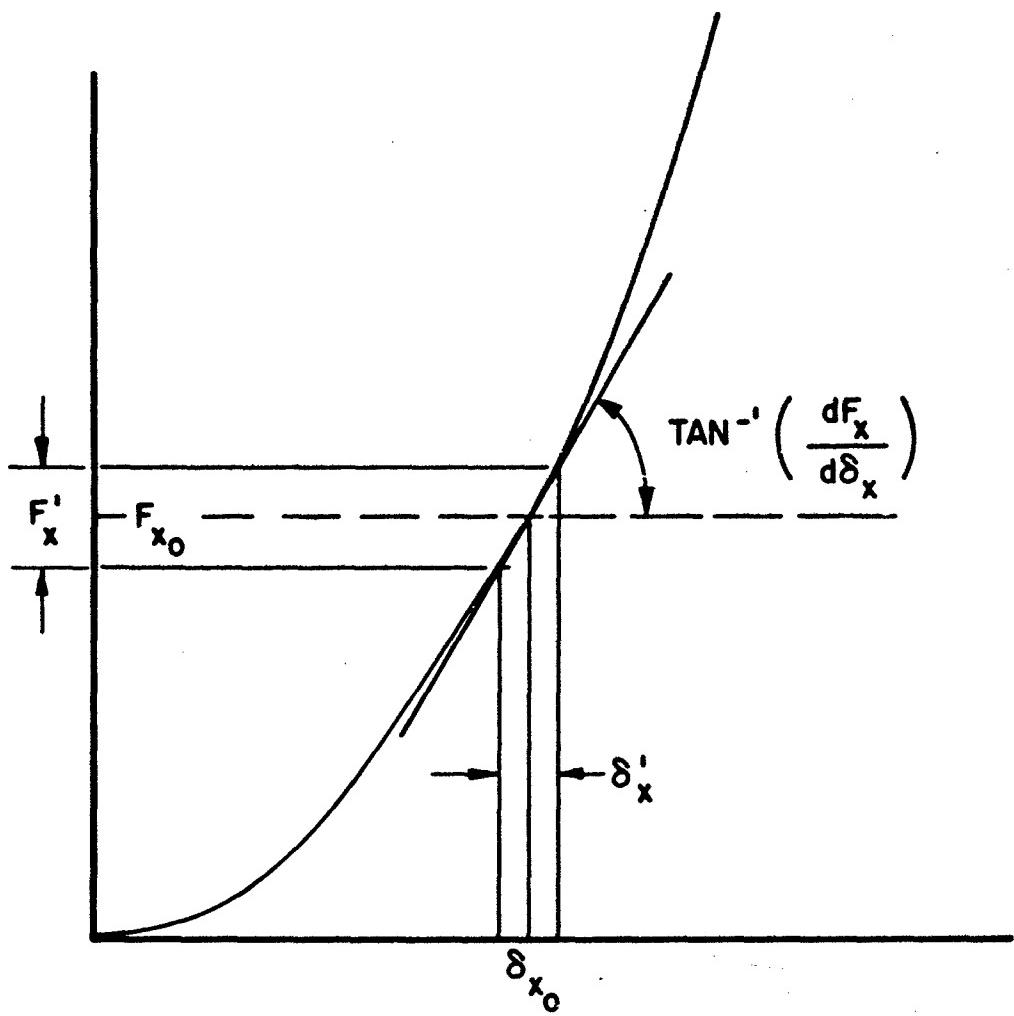


Figure 2. Linearization of Tapered Roller Bearing Stiffness

As illustrated, the load-displacement relationship is a 10/9 power law in accordance with the Hertzian point contact formula. It is not possible to describe the entire range by a linear approximation. However, if a small dynamic perturbation is taken around a static equilibrium point, $\delta'_x < \delta_{x_0}$, the small segment of the load-displacement curve can be approximated by a local tangent line. The corresponding force increment is

$$F'_x = \frac{\partial F_x}{\partial \delta_x} \delta'_x \quad (12)$$

where δ'_x is the incremental displacement. $\partial F_x / \partial \delta_x$ will depend on the amplitude of δ_{x_0} .

The question of history dependence is resolved by regarding x' as periodic motions at any frequency v of interest, and Z_{ij} accordingly would have both real and imaginary parts and may also be dependent on both the rotor speed ω and the vibration frequency v .

To avoid notational clumsiness, the primes will be dropped from (F' , x') which are understood to be dynamic perturbation quantities unless the subscript "o" is used to designate the static equilibrium condition.

2.3 Tapered Roller Bearing Characterization

In many ways the tapered roller bearing is much simpler to model from a rotor dynamic point of view than a fluid film bearing. In general, the following two simplifications can be made:

- . The restraining characteristics do not include cross coupling among the various displacement/inclination coordinates.
- . The restraining force/moment is normally temporally in phase with the displacement/inclination.

Figure 3 shows a tapered roller bearing referred to in an orthogonal xyz coordinate system. The outer ring is fixed but the inner ring may move with respect to the coordinate system. Both rings are free to rotate about their axes.

Three lineal displacements, δ_x , δ_y , δ_z , and two angular displacements, θ_x , θ_y , are required to define the spatial position and attitude of the inner ring when it is displaced from its initial position. For purposes of derivation the initial situation is that existing when the bearing's end play is just taken up in the thrust direction. Figure 4 shows these displacements in the positive sense. Figure 5 establishes the convention of the roller-position index q.

2.3.1 Stiffness

The total characterization of a tapered roller bearing's stiffness can be expressed by the matrix.

$$[K] = \begin{bmatrix} \frac{\partial F_x}{\partial x} & \frac{\partial F_x}{\partial y} & \frac{\partial F_x}{\partial z} & \frac{\partial F_x}{\partial \theta_x} & \frac{\partial F_x}{\partial \theta_y} \\ \frac{\partial F_y}{\partial x} & \frac{\partial F_y}{\partial y} & \frac{\partial F_y}{\partial z} & \frac{\partial F_y}{\partial \theta_x} & \frac{\partial F_y}{\partial \theta_y} \\ \frac{\partial F_z}{\partial x} & \frac{\partial F_z}{\partial y} & \frac{\partial F_z}{\partial z} & \frac{\partial F_z}{\partial \theta_x} & \frac{\partial F_z}{\partial \theta_y} \\ \frac{\partial M_x}{\partial x} & \frac{\partial M_x}{\partial y} & \frac{\partial M_x}{\partial z} & \frac{\partial M_x}{\partial \theta_x} & \frac{\partial M_x}{\partial \theta_y} \\ \frac{\partial M_y}{\partial x} & \frac{\partial M_y}{\partial y} & \frac{\partial M_y}{\partial z} & \frac{\partial M_y}{\partial \theta_x} & \frac{\partial M_y}{\partial \theta_y} \end{bmatrix} \quad (13)$$

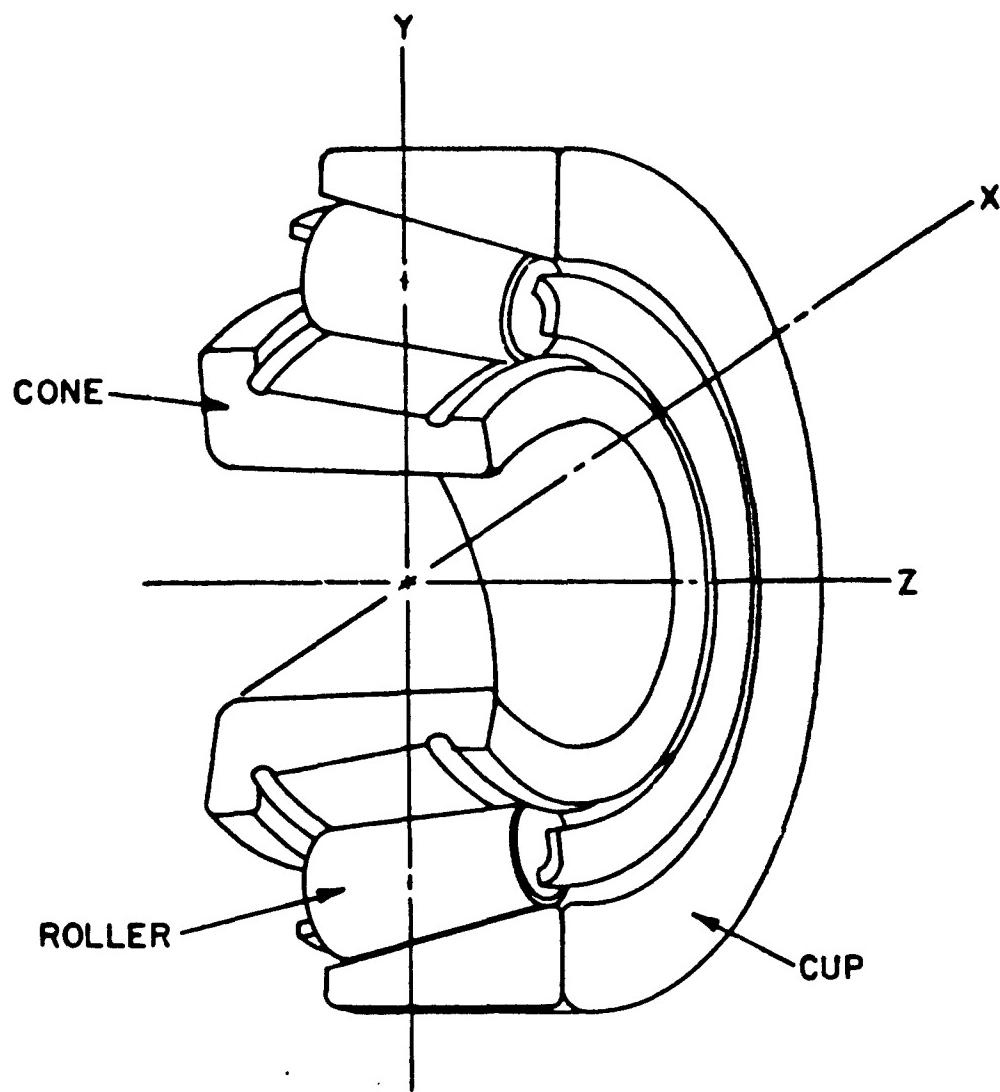


Figure 3. Tapered Roller Bearing

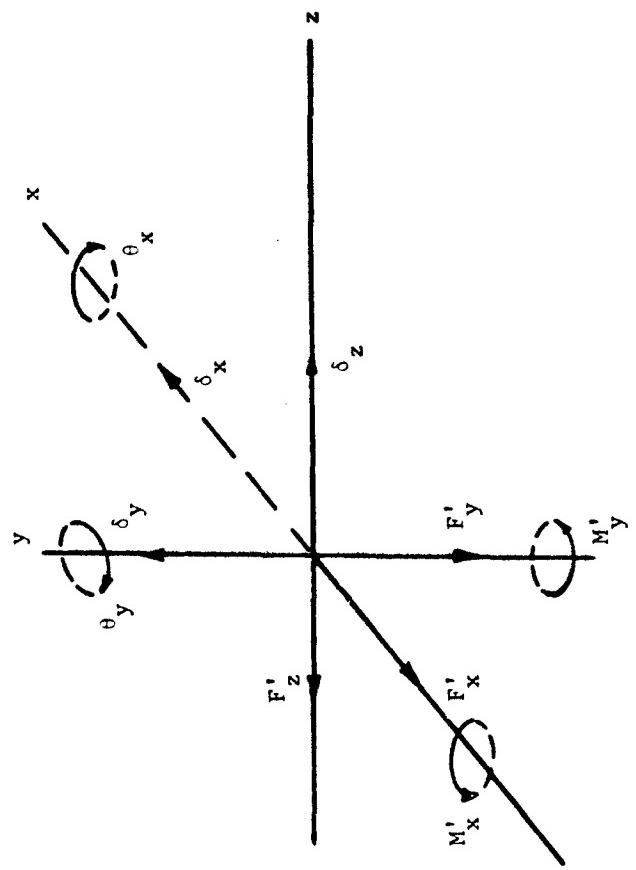


Figure 4. Bearing Coordinate System

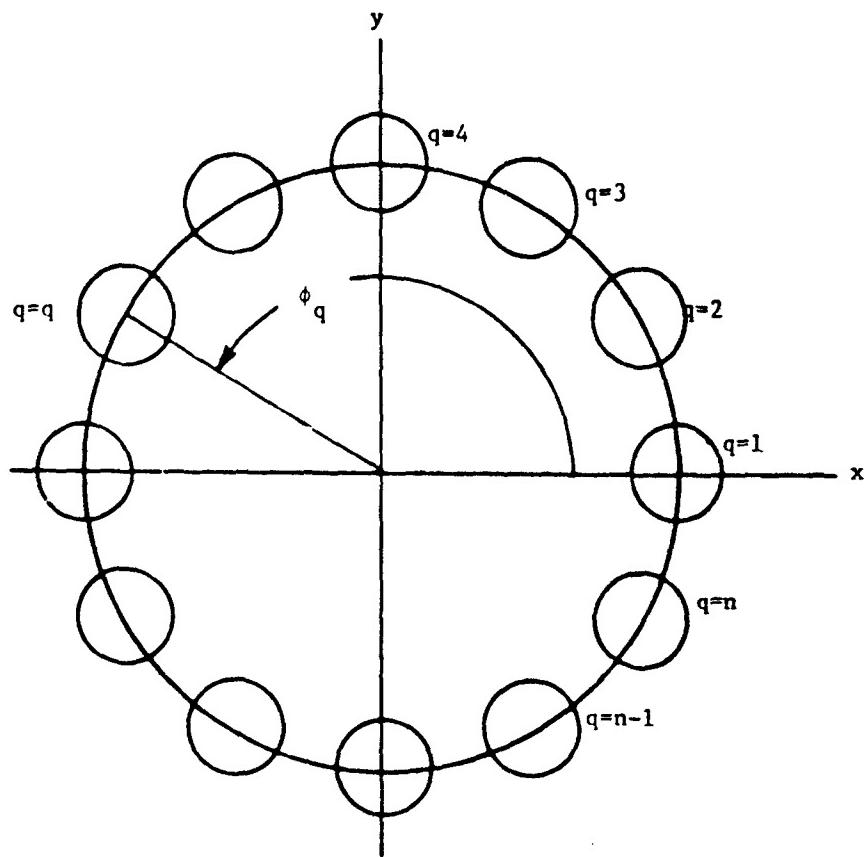


Figure 5. Tapered Roller Bearing Index, q

The lineal and angular stiffness matrices (Equations 5 and 7) can be derived from Equation (13). For example:

$$(\underline{\underline{K}})_{\text{lineal}} = \begin{bmatrix} \frac{\partial F_x}{\partial x} & \frac{\partial F_x}{\partial y} \\ \frac{\partial F_y}{\partial x} & \frac{\partial F_y}{\partial y} \end{bmatrix} \quad (14)$$

$$(\underline{\underline{K}})_{\text{angular}} = \begin{bmatrix} \frac{\partial M_x}{\partial \theta_x} & \frac{\partial M_x}{\partial \theta_y} \\ \frac{\partial M_y}{\partial \theta_x} & \frac{\partial M_y}{\partial \theta_y} \end{bmatrix} \quad (15)$$

Note that although the axial components of stiffness are not utilized by the lateral rotor dynamics program (5), they have been retained in the general tapered roller bearing stiffness matrix, Equation (13). The axial stiffness would be required, for example, if the reader was calculating the axial natural frequency of a tapered roller bearing mounted shaft.

2.3.2 Damping

An extensive search of the literature revealed no experimental damping data for tapered roller bearings. As the current state-of-the-art does not permit accurate calculation of tapered roller bearing damping, no damping data is included in this report.

2.4 Tapered Roller Bearing Under Combined Loading

Solution for the stiffness matrix of a tapered roller bearing under combined loading is a tedious problem and requires the use of a digital computer. In this section, the derivation of the solution is described. A computer program for obtaining the solution is included in the Appendix.

2.4.1 Bearing Applied Forces and Moments

As the result of the five displacements described previously in Figures 3 and 4, there are the reactions F'_x , F'_y , F'_z , and M'_x and M'_y . F'_x , F'_y , and F'_z are forces. M'_x and M'_y are moments. All are shown in their positive sense in Figure 4. External forces F_x and F_z may be applied at the inner ring center. The senses of the signs are the same as for the reactions F'_x and F'_z .

2.4.2 Roller Geometry

Figure 6 shows the boundary dimensions of a typical tapered roller. Roller mass, moment of inertia, and location of the center of gravity are calculated assuming the roller is a flat-ended, truncated cone bounded by R_1 , R_2 , and l_t .

In general, the big-end face of the roller is not flat but is a sphere having the radius R_e which is generally a proportion of the slant height, l_s , of the untruncated roller cone. Roller crown and corner breaks are also omitted from mass and moment of inertia calculations as their contributions are second order.

Figure 7 is a more complete sketch of the roller showing the details of the roller crown. The big-end spherical surface is neglected here also.

τ is the included angle of the roller cone and is obtained by iteration of

$$\frac{\tau}{2} = \tan^{-1} \left(\frac{d}{E} \sin \left(\beta - \frac{\tau}{2} \right) \right) \quad (16)$$

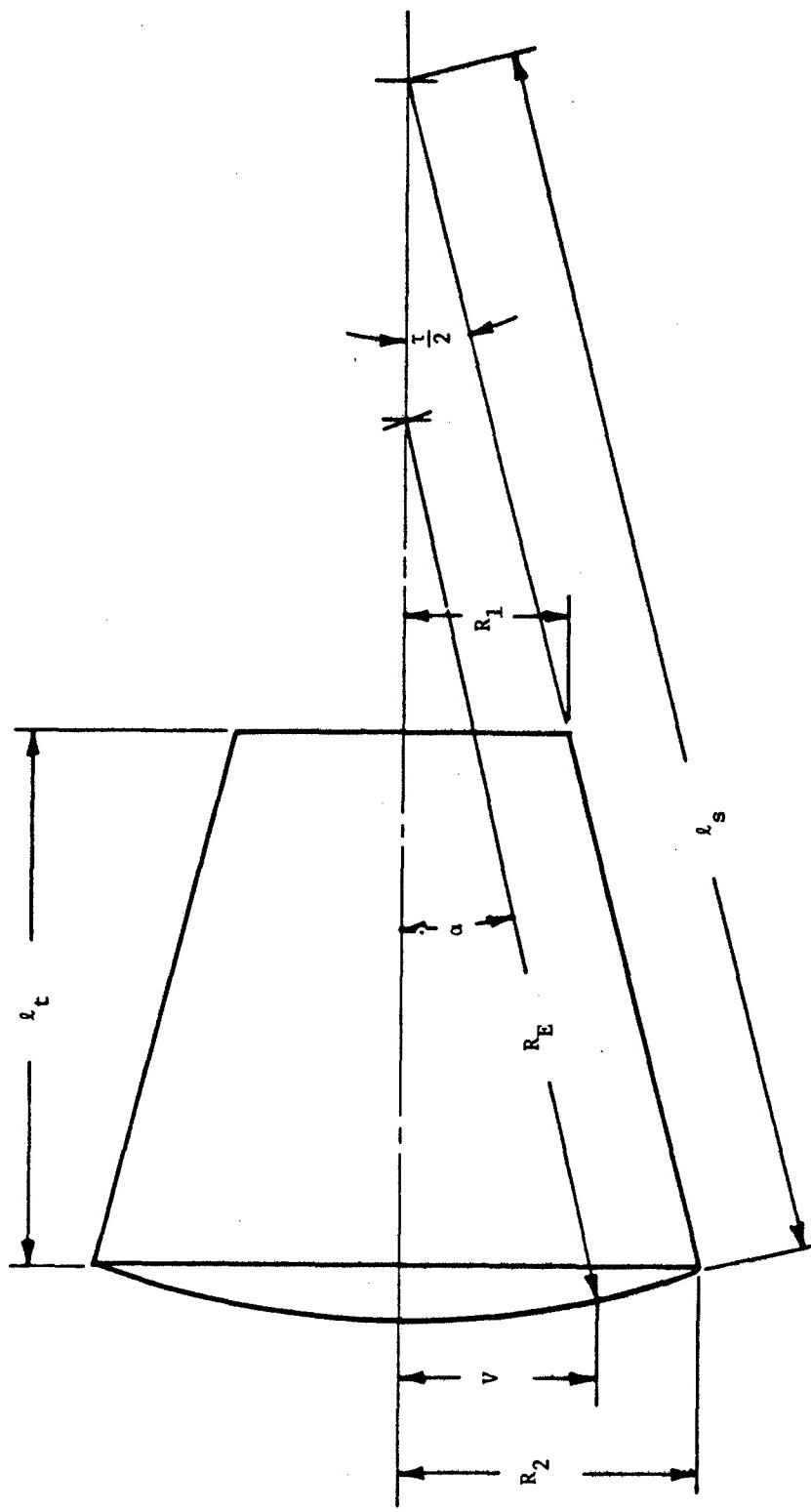


Figure 6. Boundary Dimensions of Typical Tapered Roller

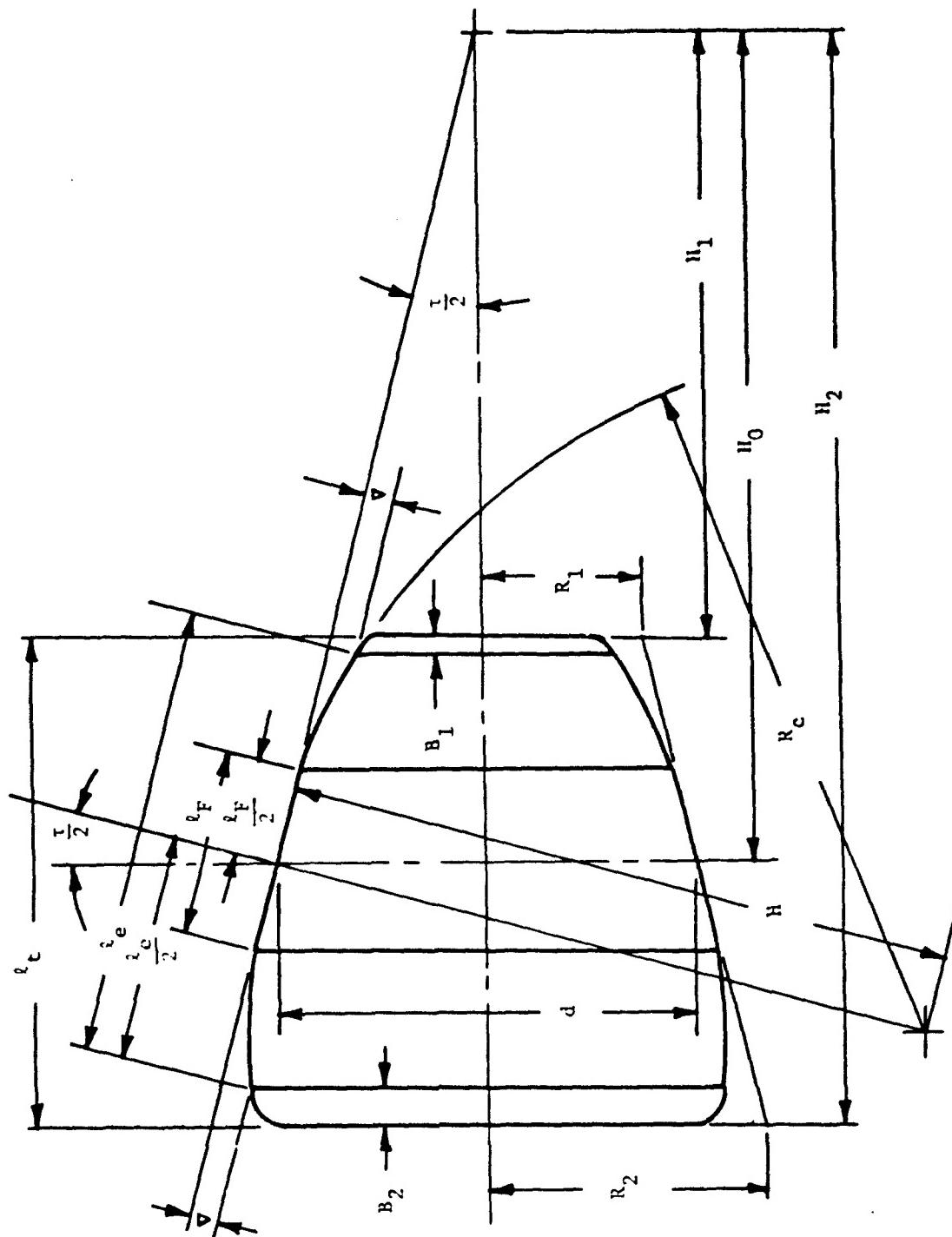


Figure 7. Dimensions of Roller Profile and Crown

From Figure 7

$$H = \sqrt{R_c^2 - \left(\frac{l_F}{2}\right)^2} \quad (17)$$

where R_c is the crown radius and l_F the length of the flat portion of the roller profile. In a fully crowned roller, the flat length is zero.

l_e is the effective length of the roller load-carrying surface. The actual working length for any loading must lie within l_e . B_1 and B_2 are the corner breaks. Their shapes are unimportant as long as they blend smoothly into the crowned surface.

∇ is the drop of the crown and is measured at the extremes of the effective length of the roller.

$$\nabla = H - \sqrt{R_c^2 - \left(\frac{l_e}{2}\right)^2} \quad (18)$$

$$H_0 = \frac{d}{2\tan\left(\frac{\tau}{2}\right)} \quad (19)$$

$$H_1 = H_0 - \frac{l_e}{2} \cos\left(\frac{\tau}{2}\right) + \nabla \sin\left(\frac{\tau}{2}\right) - B_1 \quad (20)$$

$$H_2 = H_0 + \frac{l_e}{2} \cos\left(\frac{\tau}{2}\right) + \nabla \sin\left(\frac{\tau}{2}\right) + B_2 \quad (21)$$

$$R_1 = H_1 \tan\left(\frac{\tau}{2}\right) \quad (22)$$

$$R_2 = H_2 \tan\left(\frac{\tau}{2}\right) \quad (23)$$

Let the cone corresponding to H_1 have the mass m_1 and a moment of inertia about its center of gravity $I_{1_{cg}}$.

Let the cone corresponding to H_2 have the mass m_2 and a moment of inertia about its center of gravity $I_{2_{cg}}$.

$$m_1 = \frac{\pi R_1^2 H_1 \rho}{3 \times 386.4} \quad (24)$$

$$m_2 = \frac{\pi R_2^2 H_2 \rho}{3 \times 386.4} \quad (25)$$

$$I_{1_{cg}} = \frac{3m_1}{5} \left(\frac{R_1^2}{4} + \frac{H_1^2}{16} \right) \quad (26)$$

$$I_{2_{cg}} = \frac{3m_2}{5} \left(\frac{R_2^2}{4} + \frac{H_2^2}{16} \right) \quad (27)$$

where ρ is the material density in lb/in³.

Then the distance \bar{X} from the big end of the roller at H_2 to the center of gravity of the roller is \bar{X}'

$$\bar{X}' = \frac{\frac{m_2 H_2}{4} - m_1 (H_2 - \frac{3H_1}{4})}{m_2 - m_1} \quad (28)$$

The moment of inertia I_{cg} of the tapered roller about its center of gravity at \bar{X} is

$$I_{cg} = I_{2_{cg}} + m_2 \left(\frac{H_2}{4} - \bar{X}' \right)^2 - I_{1_{cg}} - m_1 \left(H_2 - \frac{3H_1}{4} - \bar{X}' \right)^2 \quad (29)$$

Later the distance \bar{X} , being the distance left from H_0 to the center of gravity of the roller, will be required.

$$\bar{X} = H_2 - H_0 - \bar{X}' \quad (30)$$

The slant height, ℓ_s , of the truncated roller cone is shown in Figure 6 and is

$$\ell_s = \frac{R_2}{\sin(\frac{\tau}{2})} \quad (31)$$

and the big-end spherical radius, R_E , is a proportion of ℓ_s .

V is the radius from the roller centerline to the point of contact of the roller and inner-race guide flange and the flange reaction. It is directed at an angle, α , where

$$\alpha = \sin^{-1} \left(\frac{V}{R_e} \right) \quad (32)$$

The lever arm of the flange reaction about the midpoint of the working surface of the roller at H_0 is

$$l = \left[\sqrt{R_E^2 - R_2^2} - H_2 + H_0 \right] \sin \alpha \quad (33)$$

Figure 8 is an enlarged view of the race profile showing the crown drop v which is measured at a distance G from the end of the effective length. The contour is the same at both ends of the roll. If the radius R_c is known, the drop at G is

$$v' = H - \sqrt{R_c^2 - \left(\frac{l_e}{2} - G \right)^2} \quad (34)$$

If the drop is known and the radius R_c is not, the radius is

$$R_c = \sqrt{\frac{\left(\frac{l_e}{2} - G \right)^2 - \left(\frac{l_F}{2} \right)^2 - v'^2}{2v'}} + \left(\frac{l_e}{2} - G \right)^2 \quad (35)$$

2.4.3 Roller Equilibrium

Figure 9 shows the forces and moments acting on a roller which is in contact with both outer and inner races and with the inner ring guide flange.

In the following discussion, the subscripts 1 and 2 refer to the outer and inner contacts, respectively.

P_1 and P_2 are the contact loads. M_1 and M_2 are contact moments resulting from nonuniform loading along the roller's length. F_c is the centrifugal force and M_G is the gyroscopic moment. The

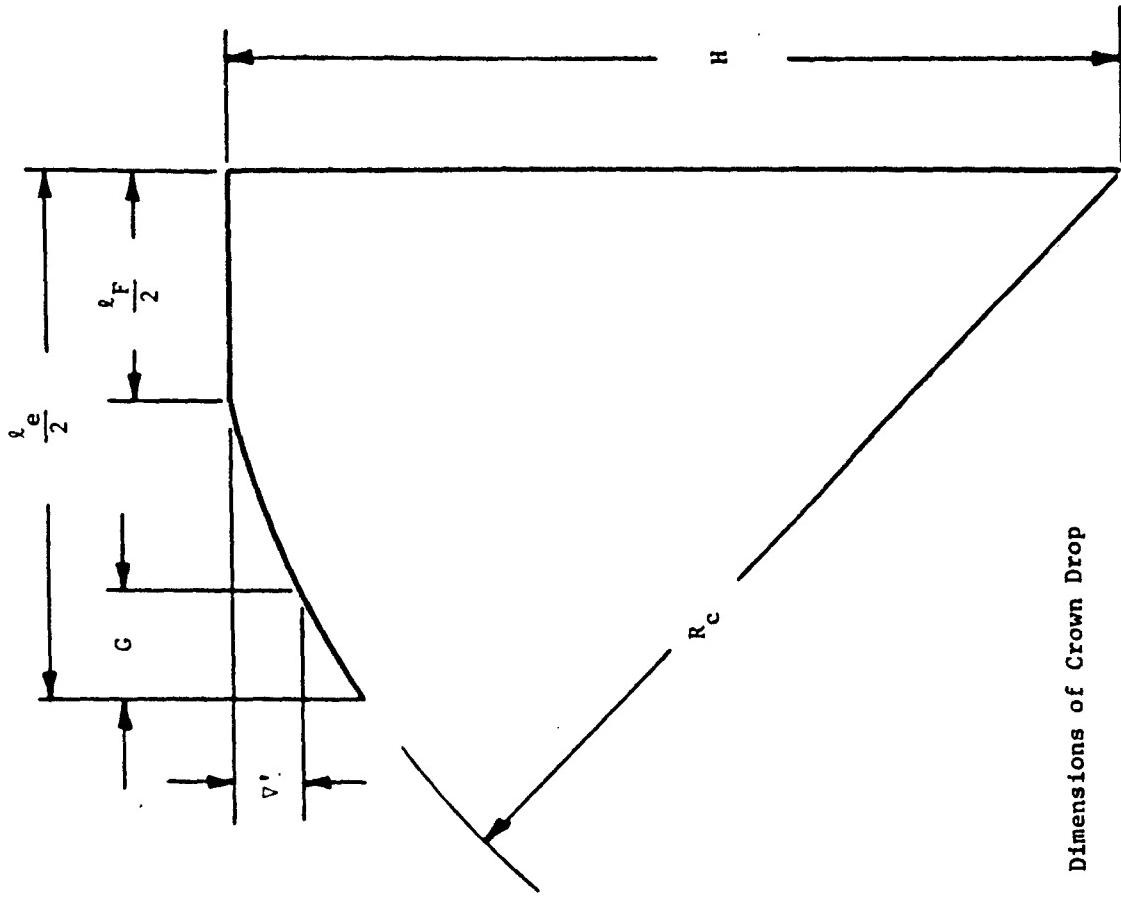


Figure 8. Dimensions of Crown Drop

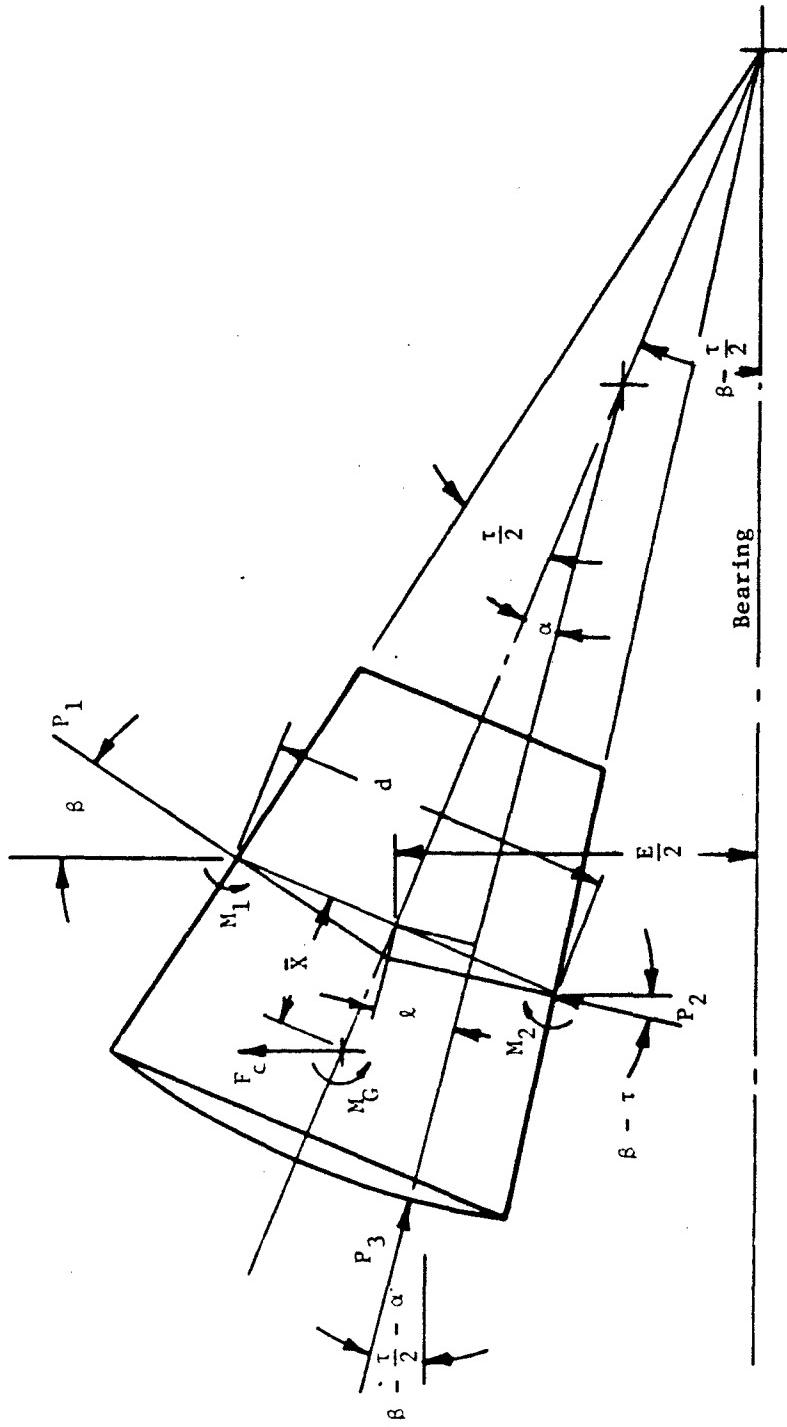


Figure 9. Forces and Moments on Roller

latter acts at the center of gravity of the roller which is located the distance \bar{x} from the central plane of the roller which contains the midpoint of the effective length.

The centrifugal force and the gyroscopic moment are

$$F_c = (m_1 + m_2) \left\{ \frac{E}{2} + \bar{x} \sin(\beta - \frac{\tau}{2}) \right\} \Omega_E^2 \quad (36)$$

$$M_G = I_{cg} \Omega_E \omega_R \sin(\beta - \frac{\tau}{2}) \quad (37)$$

where Ω_E is the orbital velocity of the roller and ω_R the angular velocity of the roller about its own center, both in radians/second.

$$\Omega_E = \frac{1}{2} \left[\Omega_1 \left(1 + \frac{d \cos(\beta - \frac{\tau}{2})}{E} \right) + \Omega_2 \left(1 - \frac{d \cos(\beta - \frac{\tau}{2})}{E} \right) \right] \quad (38)$$

$$\omega_R = \frac{E}{2d} \left[(\Omega_1 - \Omega_2) \left(1 - \frac{d \cos(\beta - \frac{\tau}{2})}{E} \right)^2 \right] \quad (39)$$

Ω_1 and Ω_2 are the input angular velocities of outer and inner rings in radians/second. P_3 is the reaction of the inner-ring flange on the roller.

In the present problem, we are concerned with external forces applied to the bearing inner ring along x and/or z (Figure 4) only. There may also be initial linear displacements along any or all of the coordinate axes, x , y , and z ; and initial rotations about x and y . These initial displacements do not change when external forces are applied along x and/or z . However, when initial rotations are present about x or z , operating displacements may occur along x and/or y as the case may be. The system, therefore, has the possibility of three degrees of freedom; i.e., working linear displacements along any or all of the axes x , y , and z . If initial displacements exist about x or y , working displacements in these modes are prevented.

The approach of the inner race to the outer race along the line defined by β for a roller at azimuth ϕ is

$$\begin{aligned}\Delta &= (\delta_z + \delta''_z) \sin\beta + \{(\delta_x + \delta''_x) \cos\phi + (\delta_y + \delta''_y) \sin\phi - \frac{P_D}{2}\} \\ &\quad \cos\beta + \frac{1}{2}\{E \sin\beta + d \sin(\frac{\tau}{2})\} \{(\theta_x + \theta''_x) \sin\phi + (\theta_y + \theta''_y) \cos\phi\}\end{aligned}\quad (40)$$

P_D is the diametral clearance or the total diametral play of the inner ring relative to the outer ring before loading.

The azimuth angle, ϕ , is related to the roller position index, q , through

$$\phi = \frac{2\pi(q-1)}{n} \quad (41)$$

where n is the number of rollers.

The double-primed items in Equation (40) are the initial displacements in the several modes.

Also, as a result of the initial misalignments which may exist about x and/or y , the inner race at the q th roller may be misaligned the amount θ .

$$\theta = (\theta_x + \theta''_x) \sin\phi + (\theta_y + \theta''_y) \cos\phi \quad (42)$$

If Δ_1 is the approach of the roller to the midpoint of the outer race, the approach Δ_2 of the inner ring to the roller at its midpoint is

$$\Delta_2 = \frac{(\Delta - \Delta_1) \cos(\alpha - \frac{\tau}{2})}{\cos(\alpha + \frac{\tau}{2})} \quad (43)$$

If θ_1 is the misalignment of the roller relative to the outer race, the misalignment θ_2 of the inner race relative to the roller is

$$\theta_2 = 0 - \theta_1 \quad (44)$$

Misalignment is positive if it tends to squeeze the big end of the roller more than the little end when the big end is at the left.

Figure 10 illustrates the geometric intersection of a roller and raceway.

The profiles of race and roller bodies are referred to an XY coordinate system. Note that the X axis is positive to the left of the origin.

The equation of the race surface is

$$Y = 0 \quad (45)$$

The equation of the flat portion of the roller or the element of the basic roller cone is

$$Y = \Delta_i + X \tan \theta_i \quad (46)$$

The equation of the crowned portion of the roller profile is

$$(X - H \sin \theta_i)^2 + (Y + H \cos \theta_i - \Delta_i)^2 = R_c^2 \quad (47)$$

The subscript i is 1 for an outer contact and 2 for an inner contact.

The intersections of the race and the crowned roller surface occur at X_{A_i} and X_{B_i}

$$X_{A_i} = \sqrt{R_c^2 - (H \cos \theta_i - \Delta_i)^2} + H \sin \theta_i \quad (48)$$

$$X_{B_i} = -\sqrt{R_c^2 - (H \cos \theta_i - \Delta_i)^2} + H \sin \theta_i \quad (49)$$

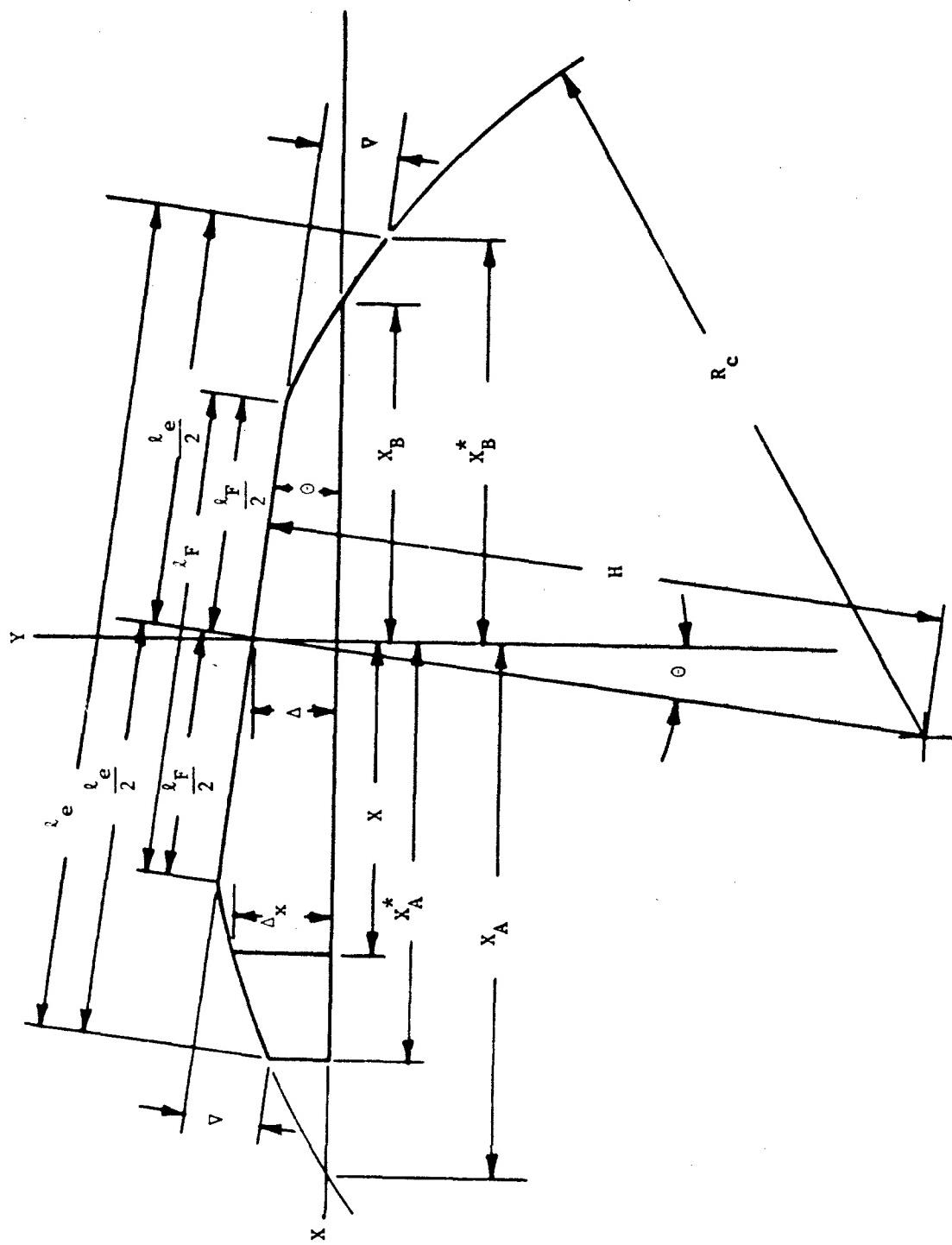


Figure 10. Geometric Intersection of a Roller and Raceway

x_{A_1} and x_{B_1} must be within the projected extremities of the roller crown. That is

$$x_{A_1} \leq x_{A_1}^* \quad (50)$$

$$x_{B_1} \geq x_{B_1}^* \quad (51)$$

where

$$x_{A_1}^* = \frac{l_e}{2} \cos\theta_i + v \sin\theta_i \quad (52)$$

$$x_{B_1}^* = -\frac{l_e}{2} \cos\theta_i + v \sin\theta_i \quad (53)$$

If the quantity under the radical in Equations (48) and (49) is zero or negative, there is no contact between roller and race.

If $\frac{l_F}{2} \cos\theta_i \geq x_{A_1}$, there is also no contact.

If $x_{A_1} > x_{A_1}^*$, x_{A_1} is set equal to $x_{A_1}^*$.

If $x_{B_1} < x_{B_1}^*$, x_{B_1} is set equal to $x_{B_1}^*$.

If $\frac{l_F}{2} \cos\theta_i > x_{B_1} > -\frac{l_F}{2} \cos\theta_i$ and $x_{A_1} > \frac{l_F}{2} \cos\theta_i$,

$$\text{the value of } x_{B_1} \text{ is } x_{B_1} = -\frac{\Delta_1}{\tan\theta_i}. \quad (54)$$

From Figure 9 the conditions for roller force equilibrium are

$$-P_1 \cos\beta + P_2 \cos(\beta-\tau) - P_3 \sin(\beta - \frac{\tau}{2} - \alpha) + F_c = 0 \quad (55)$$

$$-P_1 \sin\beta + P_2 \sin(\beta-\tau) + P_3 \cos(\beta - \frac{\tau}{2} - \alpha) = 0 \quad (56)$$

Equations (55) and (56) are a set of simultaneous nonlinear equations in which the variables are Δ_1 and θ_i at the outer contact of the

particular roller.

The flange reaction P_3 is obtained by taking moments about the roller midpoint.

$$P_3 = \frac{(-M_1 + M_2 - M_G + F_c \bar{x} \cos(\beta - \frac{\tau}{2}) - \frac{d}{2} (P_1 - P_2) \sin(\frac{\tau}{2}))}{l} \quad (57)$$

From Figure 10 the intrusion of the roller into the race is

$$\Delta_x = \Delta_i + X \tan \theta_i \quad |X| \leq \frac{l_F}{2} \cos \theta_i \quad (58)$$

$$\Delta_x = \sqrt{R_c^2 - (X - H \sin \theta_i)^2} - H \cos \theta_i + \Delta_i \quad |X| > \frac{l_F}{2} \cos \theta_i \quad (59)$$

The derivatives of Δ_x with respect to θ_i will be required later and are

$$\frac{d\Delta_x}{d\theta_i} = \frac{X}{\cos^2 \theta_i} \quad |X| \leq \frac{l_F}{2} \cos \theta_i \quad (60)$$

$$\frac{d\Delta_x}{d\theta_i} = \frac{(X - H \sin \theta_i) H \cos \theta_i}{\sqrt{R_c^2 - (X - H \sin \theta_i)^2}} + H \sin \theta_i \quad |X| > \frac{l_F}{2} \cos \theta_i \quad (61)$$

Lundberg (6) gives the approach Δ_x of two cylindrical bodies pressed together with the uniform loading p_x as

$$\Delta_x = \frac{(\eta_R + \eta_E)}{2\pi} p_x \left\{ 1.8864 + \ln \left(\frac{X_A - X_B}{2b_x} \right) \right\} \quad (62)$$

η_R and η_E are elastic constants for race and roller, respectively, having the form

$$\eta_{R,E} = \frac{4(1 - v^2)}{E_{R,E}} \quad (63)$$

where v is Poisson's Ratio and E is the modulus of elasticity.

b_x is the semi-width of the pressure area in the rolling direction.

$$b_x = \left[\frac{(n_R + n_E)}{2\pi} p_x d_x (1 + C_i \gamma_i) \right]^{1/2} \quad (64)$$

C_i is 1 for $i = 1$, corresponding to an outer contact; and -1 for $i = 2$, corresponding to an inner contact.

$$\gamma_1 = \frac{d_x \cos \beta}{E_x} \quad (65)$$

$$\gamma_2 = \frac{d_x \cos(\beta - \tau)}{E_x} \quad (66)$$

where

$$d_x = \frac{d + 2x \sin(\frac{\tau}{2})}{\cos(\frac{\tau}{2})} \quad (67)$$

$$E_x = E + 2x \sin \beta + d \cos(\beta - \frac{\tau}{2}) - d_x \cos \beta \quad (68)$$

The value of p_x corresponding to Δ_x is required. This cannot be obtained from Equation (62) in closed form. It can be obtained numerically in the following manner.

Let p'_x be an estimate of p_x . A good starting value is

$$p'_x = \frac{5 \times 10^7 \Delta_x^{10/9}}{(x_A - x_B)^{1/9}} \quad (69)$$

An improved value of p_x is

$$p_x = p'_x - \frac{(\Delta'_x - \Delta_x)}{d\Delta'_x/dp'_x} \quad (70)$$

Δ'_x is the approach of race and roller bodies calculated for the current estimate of p'_x using Equation (62).

$d\Delta'_x/dp'_x$ is obtained from Equations (62) and (64) using the current estimate p'_x and is

$$\frac{d\Delta'_x}{dp'_x} = \frac{(n_R + n_E)}{2\pi} \left\{ 1.3864 + \ln \left(\frac{x_A - x_B}{2b_x} \right) \right\} \quad (71)$$

Iteration of Equation (70) yields p_x to any desired accuracy.

The contact force, P , and the moment, M , are

$$P_i = \int_{x_{B_i}}^{x_{A_i}} p_x dx \quad (72)$$

$$M_i = \int_{x_{B_i}}^{x_{A_i}} x p_x dx \quad (73)$$

Equations (55) and (56) may now be solved for Δ_1 and θ_1 , the displacements at the outer contact. Again, a closed-form solution cannot be obtained and numerical techniques are employed.

If estimates are made of the variables Δ_1 and θ_1 , Equations (55) and (56) may not be satisfied and there will be the residues ϵ_1 and ϵ_2 for Equations (55) and (56), respectively. Differentiating Equations (55) and (56) gives:

$$\frac{d\epsilon_1}{d\Delta_1} = -\cos\beta \frac{dP_1}{d\Delta_1} + \cos(\beta-\tau) \frac{dP_2}{d\Delta_2} \frac{d\Delta_2}{d\Delta_1} - \sin(\beta - \frac{\tau}{2} - \alpha) \frac{dP_3}{d\Delta_1} \quad (74)$$

$$\frac{d\epsilon_1}{d\theta_1} = -\cos\beta \frac{dP_1}{d\theta_1} + \cos(\beta-\tau) \frac{dP_2}{d\theta_2} \frac{d\theta_2}{d\theta_1} - \sin(\beta - \frac{\tau}{2} - \alpha) \frac{dP_3}{d\theta_1} \quad (75)$$

$$\frac{d\epsilon_2}{d\Delta_1} = \sin\beta \frac{dP_1}{d\Delta_1} + \sin(\beta-\tau) \frac{dP_2}{d\Delta_2} \frac{d\Delta_2}{d\Delta_1} + \cos(\beta - \frac{\tau}{2} - \alpha) \frac{dP_3}{d\Delta_1} \quad (76)$$

$$\frac{d\epsilon_2}{d\theta_1} = \sin\beta \frac{dP_1}{d\theta_1} + \sin(\beta - \tau) \frac{dP_2}{d\theta_2} \frac{d\theta_2}{d\theta_1} + \cos(\beta - \frac{\tau}{2} - \alpha) \frac{dP_3}{d\theta_1}$$

(77)

From Equations (43) and (44)

$$\frac{d\Delta_2}{d\Delta_1} = \frac{-\cos(\alpha - \frac{\tau}{2})}{\cos(\alpha + \frac{\tau}{2})}$$

(78)

$$\frac{d\theta_2}{d\theta_1} = -1$$

(79)

And, from Equation (57)

$$\frac{dP_3}{d\Delta_1} = \frac{-\frac{dM_1}{d\Delta_1} + \frac{dM_2}{d\Delta_2} \frac{d\Delta_2}{d\Delta_1} - \frac{d}{2} (\frac{dP_1}{d\Delta_1} - \frac{dP_2}{d\Delta_2} \frac{d\Delta_2}{d\Delta_1}) \sin(\frac{\tau}{2})}{l}$$

(80)

$$\frac{dP_3}{d\theta_1} = \frac{-\frac{dM_1}{d\theta_1} + \frac{dM_2}{d\theta_2} \frac{d\theta_2}{d\theta_1} - \frac{d}{2} (\frac{dM_1}{d\theta_1} - \frac{dM_2}{d\theta_2} \frac{d\theta_2}{d\theta_1}) \sin(\frac{\tau}{2})}{l}$$

(81)

If Δ'_1 and θ'_1 are current estimates, improved estimates are:

$$\Delta_1 = \Delta'_1 - \frac{\begin{vmatrix} \epsilon_1 & \frac{d\epsilon_1}{d\theta_1} \\ \epsilon_2 & \frac{d\epsilon_2}{d\theta_1} \end{vmatrix}}{\begin{vmatrix} \frac{d\epsilon_1}{d\Delta_1} & \frac{d\epsilon_1}{d\theta_1} \\ \frac{d\epsilon_2}{d\Delta_1} & \frac{d\epsilon_2}{d\theta_1} \end{vmatrix}}$$

(82)

$$\theta_1 = \theta_1' - \frac{\begin{vmatrix} \frac{d\epsilon_1}{d\Delta_1} & \epsilon_1 \\ \frac{d\epsilon_2}{d\Delta_1} & \epsilon_2 \end{vmatrix}}{\begin{vmatrix} \frac{d\epsilon_1}{d\Delta_1} & \frac{d\epsilon_1}{d\theta_1} \\ \frac{d\epsilon_2}{d\Delta_1} & \frac{d\epsilon_2}{d\theta_1} \end{vmatrix}} \quad (83)$$

The determinants in Equations (82) and (83) are calculated at current estimates.

The derivatives of P_i and M_i with respect to Δ_i and θ_i are

$$\frac{dP_i}{d\Delta_i} = \int_{X_{B_i}}^{X_{A_i}} \frac{dp_x}{d\Delta_x} \frac{d\Delta_x}{d\Delta_i} dX \quad (84)$$

$$\frac{dP_i}{d\theta_i} = \int_{X_{B_i}}^{X_{A_i}} \frac{dp_x}{d\Delta_x} \frac{d\Delta_x}{d\theta_i} dX \quad (85)$$

$$\frac{dM_i}{d\Delta_i} = \int_{X_{B_i}}^{X_{A_i}} X \frac{dp_x}{d\Delta_x} \frac{d\Delta_x}{d\Delta_i} dX \quad (86)$$

$$\frac{dM_i}{d\theta_i} = \int_{X_{B_i}}^{X_{A_i}} X \frac{dp_x}{d\Delta_x} \frac{d\Delta_x}{d\theta_i} dX \quad (87)$$

The value of $dp_x/d\Delta_x$ is obtained from Equation (71) and the value of $d\Delta_x/d\Delta_i$ is unity.

If Equations (43), (44), (55), and (56) are differentiated with respect to Δ , there results four equations which are linear in

$d\Delta_1/d\Delta$, $d\Delta_2/d\Delta$, $d\theta_1/d\Delta$, and $d\theta_2/d\Delta$ and from which all four derivatives can be obtained. Of the four derivatives, only $d\Delta_1/d\Delta$ and $d\theta_1/d\Delta$ are of interest here.

$$\begin{aligned} & \left[-\cos\beta \frac{dP_1}{d\Delta_1} - \sin(\beta - \frac{\tau}{2} - \alpha) \frac{dP_3}{d\Delta_1} \right] \frac{d\Delta_1}{d\Delta} + \left[\cos(\beta-\tau) \frac{dP_2}{d\Delta_2} - \right. \\ & \left. \sin(\beta - \frac{\tau}{2} - \alpha) \frac{dP_3}{d\Delta_2} \right] \frac{d\Delta_2}{d\Delta} + \left[-\cos\beta \frac{dP_1}{d\theta_1} - \sin(\beta - \frac{\tau}{2} - \alpha) \frac{dP_3}{d\theta_1} \right] \\ & \frac{d\theta_1}{d\Delta} + \left[\cos(\beta-\tau) \frac{dP_2}{d\theta_2} - \sin(\beta - \frac{\tau}{2} - \alpha) \frac{dP_3}{d\theta_2} \right] \frac{d\theta_2}{d\Delta} = 0 \quad (88) \end{aligned}$$

$$\begin{aligned} & \left[-\sin\beta \frac{dP_1}{d\Delta_1} + \cos(\beta - \frac{\tau}{2} - \alpha) \frac{dP_3}{d\Delta_1} \right] \frac{d\Delta_1}{d\Delta} + \left[\sin(\beta-\tau) \frac{dP_2}{d\Delta_2} + \right. \\ & \left. \cos(\beta - \frac{\tau}{2} - \alpha) \frac{dP_3}{d\Delta_2} \right] \frac{d\Delta_2}{d\Delta} + \left[-\sin\beta \frac{dP_1}{d\theta_1} + \cos(\beta - \frac{\tau}{2} - \alpha) \frac{dP_3}{d\theta_1} \right] \\ & \frac{d\theta_1}{d\Delta} + \left[\sin(\beta-\tau) \frac{dP_2}{d\theta_2} + \cos(\beta - \frac{\tau}{2} - \alpha) \frac{dP_3}{d\theta_2} \right] \frac{d\theta_2}{d\Delta} = 0 \quad (89) \end{aligned}$$

$$\frac{d\Delta_1}{d\Delta} + \frac{\cos(\alpha + \frac{\tau}{2})}{\cos(\alpha - \frac{\tau}{2})} \frac{d\Delta_2}{d\Delta} = 1 \quad (90)$$

$$\frac{d\alpha_1}{d\Delta} + \frac{d\alpha_2}{d\Delta} = 0 \quad (91)$$

Equations (88) through (91) are easily solved for $d\Delta_1/d\Delta$ and $d\theta_1/d\Delta$. $d\Delta_1/d\theta$ and $d\theta_1/d\theta$ are obtained in a similar manner.

2.4.4 Bearing Equilibrium

The reactions of the bearing on the shaft at the central plane of the roller are

$$F'_x = \cos\beta \sum_{q=1}^n P_{1q} \cos\phi_q \quad (92)$$

$$F'_y = \cos\beta \sum_{q=1}^n P_{1q} \sin\phi_q \quad (43)$$

$$F'_z = \sin\beta \sum_{q=1}^n P_{1q} \quad (94)$$

$$M'_x = \sum_{q=1}^n \left[\frac{1}{2} \{ E \sin\beta + d \sin(\frac{\tau}{2}) \} P_{1q} + M_{1q} \right] \sin\phi_q \quad (95)$$

$$M'_y = \sum_{q=1}^n \left[\frac{1}{2} \{ E \sin\beta + d \sin(\frac{\tau}{2}) \} P_{1q} + M_{1q} \right] \cos\phi_q \quad (96)$$

Considering the three-degree-of-freedom system, the inner ring is acted upon by the external forces F_x and F_z and may have working displacements along x , y , and z . Equilibrium requires that

$$F'_x + F_x = 0 \quad (97)$$

$$F'_y = 0 \quad (98)$$

$$F'_z + F_z = 0 \quad (99)$$

Here the variables are δ_x , δ_y , and δ_z . Again, a direct solution is not possible and numerical methods must be employed.

For initial estimates δ'_x , δ'_y , δ'_z of the variables Equations (97), (98), and (99) may not be satisfied and there remain the residues ϵ_1 , ϵ_2 , and ϵ_3 . Improved values of the variables are

$$\delta_x = \delta'_x - \frac{\begin{vmatrix} \epsilon_1 & \frac{d\epsilon_1}{d\delta_y} & \frac{d\epsilon_1}{d\delta_z} \\ \epsilon_2 & \frac{d\epsilon_2}{d\delta_y} & \frac{d\epsilon_2}{d\delta_z} \\ \epsilon_3 & \frac{d\epsilon_3}{d\delta_y} & \frac{d\epsilon_3}{d\delta_z} \end{vmatrix}}{D} \quad (100)$$

$$\delta_y = \delta'_y - \frac{\begin{vmatrix} \frac{d\epsilon_1}{d\delta_x} & \epsilon_1 & \frac{d\epsilon_1}{d\delta_z} \\ \frac{d\epsilon_2}{d\delta_x} & \epsilon_2 & \frac{d\epsilon_2}{d\delta_z} \\ \frac{d\epsilon_3}{d\delta_x} & \epsilon_3 & \frac{d\epsilon_3}{d\delta_z} \end{vmatrix}}{D} \quad (101)$$

$$\delta_z = \delta'_z - \frac{\begin{vmatrix} \frac{d\epsilon_1}{d\delta_x} & \frac{d\epsilon_1}{d\delta_y} & \epsilon_1 \\ \frac{d\epsilon_2}{d\delta_x} & \frac{d\epsilon_2}{d\delta_y} & \epsilon_2 \\ \frac{d\epsilon_3}{d\delta_x} & \frac{d\epsilon_3}{d\delta_y} & \epsilon_3 \end{vmatrix}}{D} \quad (102)$$

where D is the determinant of the system.

$$D = \begin{vmatrix} \frac{d\epsilon_1}{d\delta_x} & \frac{d\epsilon_1}{d\delta_y} & \frac{d\epsilon_1}{d\delta_z} \\ \frac{d\epsilon_2}{d\delta_x} & \frac{d\epsilon_2}{d\delta_y} & \frac{d\epsilon_2}{d\delta_z} \\ \frac{d\epsilon_3}{d\delta_x} & \frac{d\epsilon_3}{d\delta_y} & \frac{d\epsilon_3}{d\delta_z} \end{vmatrix} \quad (103)$$

The right members of Equations (100) through (103) are evaluated at current estimates

$$\frac{d\epsilon_1}{d(\delta_x, \delta_y, \delta_z)} = \frac{dF'_x}{d(\delta_x, \delta_y, \delta_z)} \quad (104)$$

$$\frac{d\epsilon_2}{d(\delta_x, \delta_y, \delta_z)} = \frac{dF'_y}{d(\delta_x, \delta_y, \delta_z)} \quad (105)$$

$$\frac{d\epsilon_3}{d(\delta_x, \delta_y, \delta_z)} = \frac{dF'_z}{d(\delta_x, \delta_y, \delta_z)} \quad (106)$$

Although only the above derivatives are required in determining the equilibrium of the system, the complete matrix is required for stiffness calculations.

$$\frac{dF'_y}{d(\delta_x, \delta_y, \delta_z, \theta_x, \theta_y)} = \cos\beta \sum_{q=1}^n \cos\phi_q \frac{dP_{1q}}{d(\delta_x, \delta_y, \delta_z, \theta_x, \theta_y)} \quad (107)$$

$$\frac{dF'_y}{d(\delta_x, \delta_y, \delta_z, \theta_x, \theta_y)} = \cos\beta \sum_{q=1}^n \cos\phi_q \frac{dP_{1q}}{d(\delta_x, \delta_y, \delta_z, \theta_x, \theta_y)} \quad (108)$$

$$\frac{dF'_z}{d(\delta_x, \delta_y, \delta_z, \theta_x, \theta_y)} = \sin\beta \sum_{q=1}^n \frac{dP_{1q}}{d(\delta_x, \delta_y, \delta_z, \theta_x, \theta_y)} \quad (109)$$

$$\begin{aligned} \frac{dM'_x}{d(\delta_x, \delta_y, \delta_z, \theta_x, \theta_y)} &= \sum_{q=1}^n \left[\frac{1}{2} \{E \sin\beta + d \sin(\frac{\tau}{2})\} \frac{dP_{1q}}{d(\delta_x, \delta_y, \delta_z, \theta_x, \theta_y)} + \right. \\ &\quad \left. \frac{dM_{1q}}{d(\delta_x, \delta_y, \delta_z, \theta_x, \theta_y)} \right] \sin\phi_q \end{aligned} \quad (110)$$

$$\begin{aligned} \frac{dM'_y}{d(\delta_x, \delta_y, \delta_z, \theta_x, \theta_y)} &= \sum_{q=1}^n \left[\frac{1}{2} \{E \sin\beta + d \sin(\frac{\tau}{2})\} \frac{dP_{1q}}{d(\delta_x, \delta_y, \delta_z, \theta_x, \theta_y)} + \right. \\ &\quad \left. \frac{dM_{1q}}{d(\delta_x, \delta_y, \delta_z, \theta_x, \theta_y)} \right] \cos\phi_q \end{aligned} \quad (111)$$

where

$$\begin{aligned} \frac{dP_{1q}}{d(\delta_x, \delta_y, \delta_z, \theta_x, \theta_y)} &= \left[\frac{dP_{1q}}{d\Delta_{1q}} \frac{d\Delta_{1q}}{d\Delta_q} + \frac{dP_{1q}}{d\Theta_{1q}} \frac{d\Theta_{1q}}{d\Delta_q} \right] \frac{d\Delta_q}{d(\delta_x, \delta_y, \delta_z, \theta_x, \theta_z)} \\ &\quad + \left[\frac{dP_{1q}}{d\Delta_{1q}} \frac{d\Delta_{1q}}{d\Theta_q} + \frac{dP_{1q}}{d\Theta_{1q}} \frac{d\Theta_{1q}}{d\Theta_q} \right] \frac{d\Theta_q}{d(\delta_x, \delta_y, \delta_z, \theta_x, \theta_y)} \end{aligned} \quad (112)$$

$$\frac{dM_1}{d(\delta_x, \delta_y, \delta_z, \theta_x, \theta_y)} = \left[\frac{dM_1}{d\Delta_1} \frac{d\Delta_1}{d\Delta_q} + \frac{dM_1}{d\theta_1} \frac{d\theta_1}{d\theta_q} \right] \frac{d\Delta_q}{d(\delta_x, \delta_y, \delta_z, \theta_x, \theta_y)} +$$

$$\left[\frac{dM_1}{d\Delta_1} \frac{d\Delta_1}{d\theta_q} + \frac{dM_1}{d\theta_1} \frac{d\theta_1}{d\theta_q} \right] \frac{d\theta_q}{d(\delta_x, \delta_y, \delta_z, \theta_x, \theta_y)}$$

(113)

The derivatives of Δ_q and θ_q with respect to the inner-ring displacements are, from Equations (40) and (42)

$$\frac{d\Delta_q}{d\delta_x} = \cos\beta \cos\phi_q \quad (114)$$

$$\frac{d\Delta_q}{d\delta_y} = \cos\beta \sin\phi_q \quad (115)$$

$$\frac{d\Delta_q}{d\delta_z} = \sin\beta \quad (116)$$

$$\frac{d\Delta_q}{d\theta_x} = \frac{1}{2}\{E \sin\beta + d \sin(\frac{\tau}{2})\} \sin\phi_q \quad (117)$$

$$\frac{d\Delta_q}{d\theta_y} = \frac{1}{2}\{E \sin\beta + d \sin(\frac{\tau}{2})\} \cos\phi_q \quad (118)$$

$$\frac{d\theta_q}{d(\delta_x, \delta_y, \delta_z)} = 0 \quad (119)$$

$$\frac{d\theta_q}{d\theta_x} = \sin\phi_q \quad (120)$$

$$\frac{d\theta_q}{d\theta_y} = \cos\phi_q$$

2.4.5 Effect of Unloaded Roller

In some instances one or more rollers may be out of contact with the inner race while in contact with the outer race and the inner-ring flange. The conditions for equilibrium of such rollers are

$$-P_1 \cos\beta - P_3 \sin(\beta - \frac{\tau}{2} - \alpha) + F_c = 0 \quad (121)$$

$$-P_1 \sin\beta + P_3 \cos(\beta - \frac{\tau}{2} - \alpha) = 0 \quad (122)$$

where

$$P_3 = \frac{-M_1 - M_G - \frac{1}{2} P_1 d \sin(\frac{\tau}{2}) + F_c \bar{x} \cos(\beta - \frac{\tau}{2})}{l} \quad (123)$$

Here the variables are Δ_1 and θ_1 . Initial estimates Δ_1' and θ_1' will generally fail to satisfy Equations (121) and (122), and there will be the residues ϵ_1 and ϵ_2 .

Improved values are

$$\Delta_1 = \Delta_1' - \frac{\begin{vmatrix} \epsilon_1 & \frac{d\epsilon_1}{d\theta_1} \\ \epsilon_2 & \frac{d\epsilon_2}{d\theta_1} \end{vmatrix}}{\begin{vmatrix} \frac{d\epsilon_1}{d\Delta_1} & \frac{d\epsilon_1}{d\theta_1} \\ \frac{d\epsilon_2}{d\Delta_1} & \frac{d\epsilon_2}{d\theta_1} \end{vmatrix}} \quad (124)$$

$$\theta_1 = \theta_1' - \frac{\begin{vmatrix} \frac{d\epsilon_1}{d\Delta_1} & \epsilon_1 \\ \frac{d\epsilon_2}{d\Delta_1} & \epsilon_2 \end{vmatrix}}{\begin{vmatrix} \frac{d\epsilon_1}{d\Delta_1} & \frac{d\epsilon_1}{d\theta_1} \\ \frac{d\epsilon_2}{d\Delta_1} & \frac{d\epsilon_2}{d\theta_1} \end{vmatrix}} \quad (125)$$

The right members of Equations (124) and (125) are evaluated at current estimates. Iteration of Equations (124) and (125) yield Δ_1 and θ_1 to any desired accuracy.

The derivatives required in Equations (124) and (125) are

$$\frac{d\epsilon_1}{d\Delta_1} = -\cos\beta \frac{dP_1}{d\Delta_1} - \sin(\beta - \frac{\tau}{2} - \alpha) \frac{dP_3}{d\Delta_1} \quad (126)$$

$$\frac{d\epsilon_1}{d\theta_1} = -\cos\beta \frac{dP_1}{d\theta_1} - \sin(\beta - \frac{\tau}{2} - \alpha) \frac{dP_3}{d\theta_1} \quad (127)$$

$$\frac{d\epsilon_2}{d\Delta_1} = -\sin\beta \frac{dP_1}{d\Delta_1} + \cos(\beta - \frac{\tau}{2} - \alpha) \frac{dP_3}{d\Delta_1} \quad (127)$$

$$\frac{d\epsilon_2}{d\theta_1} = -\sin\beta \frac{dP_1}{d\theta_1} + \cos(\beta - \frac{\tau}{2} - \alpha) \frac{dP_3}{d\theta_1} \quad (128)$$

where

$$\frac{dP_3}{d\Delta_1} = \frac{-\frac{dM_1}{d\Delta_1} - \frac{1}{2} d\sin(\frac{\tau}{2}) \frac{dP_1}{d\Delta_1}}{\ell} \quad (129)$$

$$\frac{dP_3}{d\theta_1} = \frac{-\frac{dM_1}{d\theta_1} - \frac{1}{2} d\sin(\frac{\tau}{2}) \frac{dP_1}{d\theta_1}}{\ell} \quad (130)$$

Rollers which are out of contact with the inner race must be considered in evaluating the bearing's reactions. They, however, contribute nothing to the stiffness matrix since P_1 and M_1 for these rollers do not change with changes in the inner-ring displacements.

SECTION III

APPLICATION OF COMPUTER PROGRAM

The analysis of Section II has been programmed in Fortran IV for a digital computer and is suitable for use on the CDC 6600. A program listing is presented in the Appendix.

3.1 Sample Test Case

To illustrate a typical case consider the bearing in Figure 11. This is a tapered roller bearing assembly modified for high speed operation. The geometry of this sample bearing is summarized below.

Number of rollers	37
Roller diameter at midpoint	.2913 in.
Pitch diameter	5.0 in.
Contact angle at outer race	14°40'
Effective length of roller	.6001 in.
Roller big-end spherical radius	0.8 in.
Radius from roller centerline to point of big-end spherical surface with inner race flange	0.75 in.
Roller crown radius	100 in.
Roller small-end corner break	.02 in.
Roller big-end corner break	.03 in.
Crown drop gage point	.03 in.

The operating conditions for the sample case are:

Rotational speed = 20,000 rpm

Load Condition #1

Thrust Load = 3,000 lbs.

Load Condition #2

Thrust Load = 3,000 lbs.

Radial Load = 700 lbs.

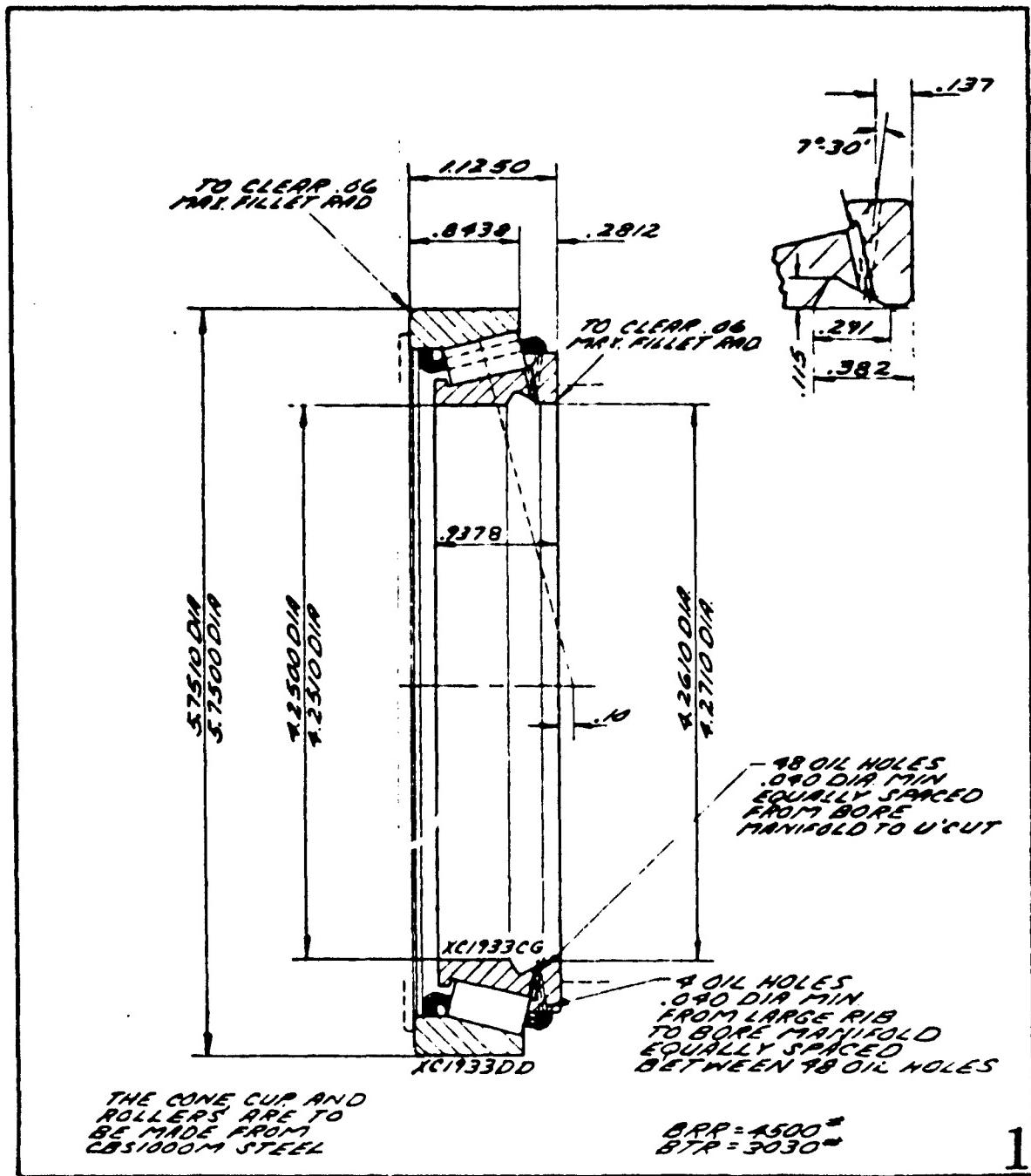


Figure 11. Sample Tapered Roller Bearing Assembly

3.2 Input Format

Figure 12 presents the input data format and Figure 13 shows the actual input data for Load Conditions #1 and #2 of the sample case.

3.3 Output Format

Figure 14 presents the output data for Load Condition #1. The input data are summarized in Figure 14, followed by the output data including the internal load distribution as well as various other stress and displacement parameters. The stiffness matrix is given on the last page of Figure 14.

The output data for Load Condition #2 are presented in Figure 15.

Number of Rolls	Roll Diameter at Midpoint - Inches	Pitch Diameter - Outer Race - Inches	Contact Angle at Outer Race - Degree (Must be Positive)	Total Length of Roll - Inches	Effective Length of Roll - Inches	Length of Flat Working Surface - Inches	Roll End Surface Spherical Radius - Inches
TITLE							
Radius from Roll Centerline to Point of Big End Sphere	Roll Crown Radius - Inches	Roll Crown Drop - Inches	Roll Small End Corner Break Inches	Crown Drop Gage Point - Inches	Material Clearance Inches	Roll Material Density lb/in ³	Roll Material
TITLE							
Surf with Inner Race Flange	Modulus of Elasticity Inner Ring - lb/in ²	Modulus of Elasticity Rolls - lb/in ²	Poisson's Ratio Inner Ring	Poisson's Ratio Rolls	Initial Displacement Along z - Inches	Initial Displacement Along z - Inches	Initial Displacement About x - Radians
Modulus of Elasticity Outer Ring - lb/in ²	RPM - Inner Ring	Force Along x - lb	Force Along x - lb	Force Along y - lb	Initial Displacement Along x - Inches	Initial Displacement Along y - Inches	Initial Displacement About y - Radians
A.1. In these fields permit a workable displacement							
Initial Displacement About y - Radians	Along x	Along y	Along z				
If blank the initial displacement is maintained							

E 10.0 Format

- (A) If total length is given omit effective length
- If effective length is given omit total length
- Enter 1 to start printout at top of new page
- (B) If drop is given omit crown drop.
- If crown radius is given omit drop.
- To run additional load cases with same bearing, repeat cards 6 and 7 directly after last card 7.
- To run new system place 2 blanks after last card 7 and repeat card 1, etc.

Figure 12. Input Data Format

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A 3000-year-old
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..

FORTNIGHTLY STATEMENT							
37	2913159	5	14 .66667				.6901
	SAMPLE PROBLEM - TAPERED ROLLER BEARING						
	FULLY-CROWNED ROLLS						
75	100.		.02				.03
	BLANK CARD						
	20000.			-3000.			
	20000.		700.		-3000.		
	1.			1.			
	BLANK CARD						
	BLANK CARD						
	BLANK CARD						

Figure 13. Sample Problem Data Input

SAMPLE PROBLEM - TAPERED ROLLER BEARING

FULLY-COOLED ROLLS

DESIGN DATA FOR BEARING NO. 1

NO. OF ROLLS	DIA METER IN	PITCH DIAMETER IN	CONTACT ANGLE DEG	TOTAL LENGTH IN	EFFECTIVE LENGTH IN	FLAT OF % IN	CROWN OF % IN	CROWN RADIUS IN
3.7400-C1	2.9132-01	5.0000-01	1.4037+01	6.5000+01	6.3010-01	0.0000-02	3.0000-02	1.0000-02
VALVE SPHERICAL INCLINATION OF END RADIUS IN	1.1272-01	8.6699+00	1.6600-00	1.1754-02	1.2277-02	1.2865-06	1.8710-03	1.8710-02
OUTER CIRCUMFERNAL INNER CLEARANCE IN	2.5000-C1	2.5000-C1	2.5000-01	2.5000-01	2.5000-01	0.0000	2.9000-C1	2.9000-07

VALVE SPHERICAL INCLINATION OF END RADIUS IN	OUTER ROLLER DIA IN	INNER ROLLER DIA IN	ROLLS PER INCH	ROLL DIA IN	ROLL LENGTH IN	ROLL DENSITY LB/IN ³	OUTER ROLLER DIA IN	OUTER ROLLER LENGTH IN	OUTER ROLLER DENSITY LB/IN ³
1.1272-01	8.6699+00	1.6600-00	1.4037+01	6.5000+01	6.3010-01	0.0000-02	3.0000-02	1.0000-02	1.0000-02

Figure 14. Output Data for Load Condition #1

Figure 14. (Continued)

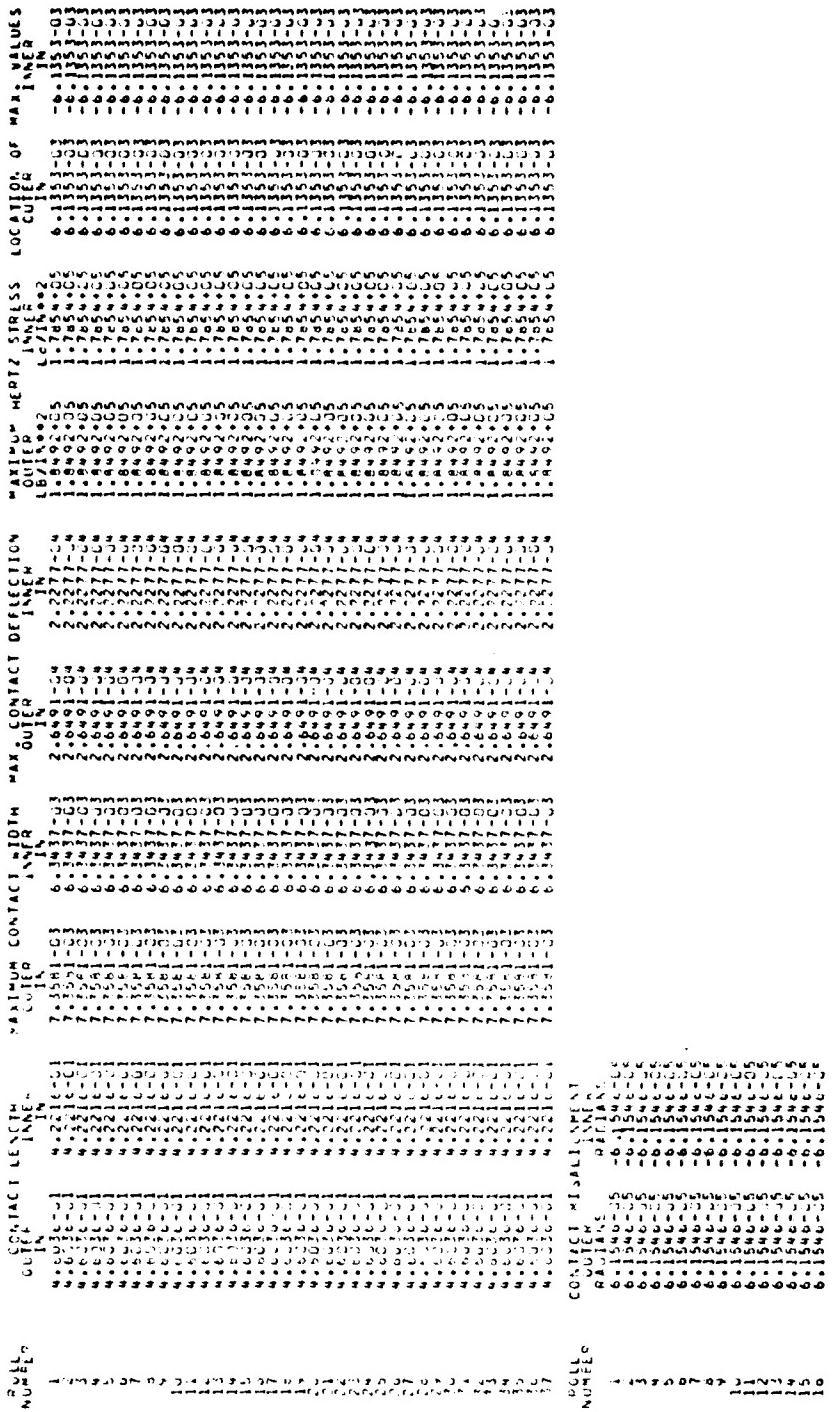


Figure 14. (Continued)

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Figure 14. (Continued)

Figure 15. Output Data for Load Condition #2

Figure 15. (Continued)

PARTIAL DERIVATIVES OF REACTIONS wITH RESPECT TO DISPLACEMENTS			
UFX/IN	DFX/DT	DFY/IN	DFY/DT
Lx/IN	Lx/DT	Lx/IN	Lx/DT
1.5488+C7	5.498C-C2	-1.5784+C6	4.2689-C2
DFZ/IN	DFY/IN	DFY/IN	DFY/DT
LBZ/IN	LBZ/DT	LBZ/IN	LBZ/DT
6.1777-C2	1.5497-J7	0.4901	1.5153-Q7
DFZ/Y	DFZ/Y	DFZ/Y	DFZ/DT
LBZ/IN	LBZ/IN	LBZ/IN	LBZ/DT
-1.5774-L5	1.54863-J3	2.1422+C6	1.9531-Q2
DFZ/Y	DFZ/Y	DFZ/Y	DFZ/DT
LBZ/IN	LBZ/IN	LBZ/IN	LBZ/DT
7.3242-C2	1.5133-J7	3.C6-C2	6.9531-Q6
DFZ/Y	DFZ/Y	DFZ/Y	DFZ/DT
LBZ/IN	LBZ/IN	LBZ/IN	LBZ/DT
1.5125-C7	1.5156-J1	-1.3551-L5	4.2440-Q2

Figure 15. (Continued)

APPENDIX

**COMPUTER PROGRAM
FOR
CALCULATING STIFFNESS MATRIX
OF
TAPERED ROLLER BEARING**

U.T.N. PROGRAM		
U.CARD	COL.	
1	1-FN 1-1U 11-2U 41-5U 21-3U 31-4U 41-5U 51-6U	NUMBER OF ROLLS - 60 UNTIL ROLL DIAMETER - IN. MEASURED AT "CROWN" OF EFFECTIVE LENGTH. SEE COLS. 51-60 OF THIS CARD. PITCH DIAMETER - IN CONTACT ANGLE AT OUTER RACE - NEG. MUST BE POSITIVE TOTAL LENGTH OF ROLL - IN. THE AXIAL DISTANCE BETWEEN THE INTERSECTION OF THE ROLL-END FACES AND THE ROLL-ONE ELEMENT MEASURED BETWEEN SHARP CORNERS EFFECTIVE LENGTH OF ROLL - IN. THE MAXIMUM WORKING LENGTH OF THE ROLL MEASURED ALONG THE ROLL CONE ELEMENT BETWEEN CORNER BREAKS
NOTE	IF TOTAL LENGTH IS GIVEN OUT EFFECTIVE LENGTH IF EFFECTIVE LENGTH IS GIVEN OUT TOTAL LENGTH	
61-7U	LENGTH OF FLAT PORTION OF ROLL WORKING SURFACE MEASURED ALONG THE CONE ELEMENT - IN. FOR A FULLY-CROWNED ROLL THE FLAT LENGTH IS ZERO	
71-8U	PULL BIG-END SURFACE SPHERICAL RADIUS - IN. IF NEGATIVE THIS ABSOLUTE VALUE MULTIPLIES THE ROLL CONE SLANT HEIGHT MEASURED FROM THE APEX TO THE SHARP INTERSECTION WITH THE BIG-END SURFACE TO GIVE THE SPHERICAL RADIUS	
?	PUNCH 1. TO START THE PRINTOUT AT THE TOP OF A NEW PAGE 2-R0 1 2-R0 1 2-R0 4	
	LEAVE BLANK TITLE CARD. PUNCH ANYTHING RADIUS FROM ROLL CENTERLINE TO POINT OF CONTACT OF BIG- END SPHERICAL SURFACE WITH THE INNER-RACE FLANGE - IN. IF NEGATIVE ITS ABSOLUTE VALUE MULTIPLIES THE BIG-END RADIUS FROM THE ROLL CENTERLINE TO THE SHARP INTERSECTION OF THE BIG-END SURFACE AND THE PROJECTED ROLL CONE ELEMENT TO GIVE THE DESIRED RADIUS	
11-2U 21-3U	ROLL CROWN RADII - IN. ROLL CROWN DROP - IN. MEASURED FROM GAGE POINT. SEE COLS. 51-60 OF THIS CARD	
NOTE	IF CROWN RADIUS IS GIVEN OUT CROWN DROP. IF DROP IS	

GIVEN OMIT CROWN RADIUS

- 31-4U ROLL SMALL-END CORNER BREAK - TN. VIEW ROLL WITH AXIS
HORIZONTAL AND WITH BIG END AT LEFT. SMALL-END CORNER
BREAK IS THE AXIAL DISTANCE BETWEEN THE ROLL-CONE
INTERSECTION OF THE SMALL END-SURFACE WITH THE ROLL-CONE
ELEMENT AND THE RIGHTMOST POINT OF THE EFFECTIVE LENGTH
ROLL PIG-END CORNER BREAK - TN. VIEW ROLL WITH AXIS
HORIZONTAL AND BIG END AT LEFT. PIG-END CORNER BREAK IS
THE AXIAL DISTANCE BETWEEN THE SHARP INTERSECTION OF THE
PIG-END SPHERICAL SURFACE WITH THE ROLL-CONE ELEMENT AND
THE LEFTMOST POINT OF THE EFFECTIVE LENGTH
CROWN DROP GAGE POINT - IN. REFERENCE POINT FOR THE
MEASUREMENT OF CROWN DROP - TN. IT IS THE DISTANCE
MEASURED ALONG THE PROJECTED ROLL-CONE ELEMENT FROM THE
OUTER EDGE OF THE EFFECTIVE LENGTH TO THE POINT OF CROWN
DROP MEASUREMENT. IT IS THE SAME FOR BOTH ENDS OF THE
ROLL
- 51-6U DIAMETRAL CLEARANCE - TN. THE TOTAL DIAMETRAL LOOSENESS
OR SHAKE IN THE MOUNTED BEARING BEFORE LOADING.
NEGATIVE VALUE INDICATES TIGHTNESS.
- 71-8U ROLL MATERIAL DENSITY - LB/IN**3
IF BLANK PROGRAM ASSUMES .283
- 5 1-10 MODULUS OF ELASTICITY FOR OUTER RING - LR/IN**2
11-2U SAME FOR INNER RING
21-3U SAME FOR ROLLS
- 31-4U PUSSON'S RATIO FOR OUTER RING
IF BLANK PROGRAM ASSUMES .25
- 41-5U SAME FOR INNER RING
51-6U SAME FOR ROLLS
- 6 1-10 RPM OF OUTER RING
11-2U RPM OF INNER RING
21-3U FORCE ALONG X - LR. CROSSIVE SIGN
31-4U FORCE ALONG Y - LB. MUST BE NEGATIVE
41-5U INITIAL DISPLACEMENT ALONG X - TN
51-6U INITIAL DISPLACEMENT ALONG Y - TN
61-7U INITIAL DISPLACEMENT ALONG Z - TN

7 71-AU INITIAL DISPLACEMENT AMOUNT X = 0.0
 1-10 INITIAL DISPLACEMENT AMOUNT Y = 0.0
 11-20 A 1. HERE PERMITS OPERATING DISPLACEMENTS ALONG Y
 21-30 A 1. HERE PERMITS OPERATING DISPLACEMENTS ALONG X
 31-40 A 1. HERE PERMITS OPERATING DISPLACEMENTS ALONG Z

TO RUN ADDITIONAL LOAD CASES WITH THE SAME BEARING REPEAT CARDS 6 AND 7 AS A UNIT DIRECTLY AFTER LAST CARD 7

TO RUN NEW SYSTEM PLACE TWO BLANKS AFTER LAST CARD 7 AND REPEAT CARDS 1 ET SEQ.

TO STOP PLACE THREE BLANKS AFTER LAST CARD 7

COMMON ALPHA,A(5,5),A(5,5),ABIG,
 1BT2AL,B6TWP,B9(2,60), C(2),CROWN,CBTA,CTAU02,CMNT,U,CRWT02,
 2CAT2AL,CP1,COOR(5),CTH,COR(5), D,DRND,DELU1(5),DFL,11(5),DFL(3)
 1,DNTV(5,5),DELT,DEL(2),DPDEL(2),DNDEL(2),OPTH(2),NTH(2),
 4DNTH,DX,DLX(2,60),DELSAV(2,60),N1D1(5),N1D1(5),NELL,DELLX,
 SE,EL(2),ER(5),EFL,EX,ER(5), FLT,FLTn,FRFE(2),FC,FAGE,SAH,
 AGAM,GM,GM,
 H1,H2,HERTZ(2,60),HR7,WN,TBR,TSTOP,ILO,D,IOUIT,
 7ICT,JPASS,
 KKK,N,NOLOAD,OFE,ONP,P1OUT,P3OUT,
 AP(2,60),P3,PX3(60),P1OUT,P3OUT,
 R5,PH,RW,R1,R2,RPN(2)
 COMMON SATA,STAUN2,SLANT,SBWTAU,SBT2AL,SAV1,SAV2,SAV3,SPH,STH,
 1,THD1(5),TAU02,TAU,TTAU02,TAT2AL,TOL(5),THETA,TH(2),TANTH,
 2THSAV(2,60),THET,TMPBTG,V,UV,WEIGHT,XN,XLT,XLE,XTAU,
 3XLTO2,XLEO2,XNAB,XMASS1,XMASS2,YMASS,XINCG1,XINCG2,XINC,G,YBARI,
 4XQAR2,XRAR,XLEVER,XALPHA,YF(2),YF1(5),XU(2),XTH,YNEL,X1,X2,
 SXWM(2,60),XXX(2,2,60),XWQAV(2,60),XWQAV(2,60),XWQAV(2,60),
 6Y'R
 DOUBLE PRECISION BGTWP,CTH,DELTA,DEL,DRNFD,DRNFT,OPTH,CMTH,GMX,HO
 1,DX,P3,ST4,THFTA,TH,TANTH,VV,XM,YTH,XTEL
 IAR=0
 C(1)=1.
 C(2)=-1.

```

10 READ(5,20)XN,D,E,RETA,XLT,XLE,FLT,RS
20 FORMAT(AE10.0)
1E(XN,E0.0)STOP
PFAD(5,30)
30 FORMAT(AN1
1      )
      WRITE(6,30)
      READ(5,20)V,CROWN,DROP,R1,B2,GAGE,PD,RHN,YM(1),YV(2),YMR,PR(1),PR(
12),PRR
      ICT=0
      ISTOP=U
      INRIBR+1
      ILOADEU
      BTA=BE1A/57.29578
      SRTAESIN(RTA)
      CRTAECS(SRTA)
      DELDI(1)=SRTA
      T4D1(1)=0.
      T4D1(2)=0.
      T4D1(3)=0.
      N=XN
      YMR=29.56
      PRR=.25
      DC 40 K=1,2
      YV(K)=29.56
      PR(K)=.25
      EL(K)=.636619A*((1.-PRR**2)/YMR+(1.-PR(V)**2)/YV(K))
      IF(RHO.FG.U.)RHC=.2A3
      GAV=D/E
      TAU02=U.
      20 50 ITT=1,3C
      TEMP=TAU02
      TAU02=ATAN(GAV*SIN(BTA-TAU02))
      IF(CRS(TAU02-TEMP)-5.E-7)70,50,50
      CONTINUE
      50
      WRITE(6,60)
      60 FORMAT(18HU MAIN PROGRAM 60)
      ISTOP=1

```

```

70 TO 160
    T:U=2.*TAU02
    YTAU=57.29579*TAU
    STA02=S1*(TAU02)
    CTA02=C05*(TAU02)
    TTAU02=CTAU02/CTAU02
    T=(XLT)*0.00.00
    XLT*XL-E*CTAU02+R1+32
80 XLE=(XLT-aE-R1)/CTAU02
    G TO 160
    XLE=(XLT-aE-R1)/CTAU02
    YLT02E=.5*YLT
    XLE02E=.5*XLE
    FLT02E=.5*FLT
    CALL PLALC(CROWN, DROP, FLT02, GAGE, HC, XI, EN)
P'H'E'H
    X'JABERMH-SQRT(CROWN**2-XLF02**2)
H0=5*J/TTAU02
H1=5*J/TTAU02-XLE02*CTAU02+XNAR*STA02-R1
H2=5*J/TTAU02+XLE02*CTAU02+XNAR*STA02+R2
R1=H1*I TAU02
R2=H2*I TAU02
    X'ASS1=2.*710139F-3*R1**2*H1*R40
    X'ASS2=2.*710139E-3*R2**2*H2*R40
    X'ASS=XMASS2-YMASS1
    XINC61=6.*XMASS1*(R1**2/4.+H1**2/16.)
    XINC62=6.*XMASS2*(R2**2/4.+H2**2/16.)
    XPAR1=H1/4.
    XPAR2=H2/4.
    XPAR=(XMASS2*XPAR2-XMASS1*(XLT+YRAR1))/YMASS
    XINC6=XINC2+YMASS2*(YBAR2-XRAR)*2-XINC61-XMASS1*(XLT+XB'R1-XBAR)
1*2
    XNARE=H2-XPAR-H0
    SLANT=R2/STA02
    TF(V.LT.0.)V=ARS(V)*R2
    IF(IRS.LT.0.)RS=ABS(RS)*SLANT
    ALFA=ASIN(V/R5)
    ALPHA=ALPHA*57.29578
    XLEVER=(H0-H2+S0RT(RS**2-R2**2))*V/RS
    RNTAU=B7A-TAU

```

```

SQRTAU=SIN(BMTAU)
CRWTAU=COS(BMTAU)
BMT02=BTA-TAU02
SMT02=SIN(BMT02)
CRMTO2=COS(BMT02)
GAMM=CBWTO2*GAM
BT2ALE=BWT02-ALPHA
SRT2ALE=SIN(BT2AL)
CRT2ALE=COS(BT2AL)
TRT2ALE=SBT2AL/CRT2AL
WEIGHT*XMASS*186.4
TOL(1)=5.E-7
TOL(2)=5.E-7
TOL(3)=1.E-6
TOL(4)=1.E-6
TOL(5)=1.E-6
WRITE(6,110)I=R
110 FORMAT(30H0 DESIGN DATA FOR BEARING NO.,I3)
WRITE(6,120)
120 FORMAT(129H0 NO. OF ROLL PITCH CONTACT TOT
1AL EFFECTIVE FLAT VALUF VALUF CROWN TOT
2 CROWN/129H ROLLS DIAMETER ANGLE L
3 LENGTH LENGTH OF B1 OF B2 RADIUS
4 DROP/18X.2HIN.10X.2HIN.9X.7HCEG .7(12H IN ))
WRITE(0,130)XN,D,E,BETA,XLT,XLE,FLT,R1,R2,CROWN,PROP
130 FORMAT(1P11E12.4)
WRITE(6,140)
140 FORMAT(125H0 VALUE SPHERICAL INCLUDED VALUE OF LOCATI
1ON OF ROLL MOM. OF IN. ROLL MODULUS OF ELASTI
2CITY/129H OF V END RADIIIS ROLL ANGLE ALPHA CENTRO
3ID WEIGHT ABOUT C.G. DENSITY OUTER INNER
4 ROLLS/131H IN DEG DEG I
5N LA LB*IN*SEC*#? LB/IN*#? LB/IN*#?
6 LB/IN*#?
WRITE(0,130)V,RS,XTAU,XALPHA,XRAP,WIGHT,YINC6,PH0,YU(1),YU(2),YVR
WRITE(6,150)
150 FORMAT(47H0 INNER ROLLS POISSON'S RATIO DIAMETRAL/4TH O
        OUTER CLEARANCE/42X,2HIN)
        WRITE(0,130)PR(1),PR(2),DRR,PD

```

```

160 READ(5,20)RPM(1),RPM(2),XF(1),XF(2),DFL11(2),DFL11(3),DFL11(1),
      DFL11(4),FL11(5),FREE(2),FREE(3),FREE(1)
      IF(IARS(RPM(1))+ABS(RPM(2)).EQ.0.160 THEN
      IF(IISTUP(ST.0).GO TO 160
      LOAD=LOAD+1
      IQUIT=0
      TEMP1=.005*D/SRTA
      TEMP2=.005*D/CRTA
      DFL(1)=0.
      DFL(2)=0.
      DFL(3)=0.
      IF(XF(1).NE.0.)DFL(1)=-SIGN(TEMP1*XF(1))
      IF(XF(2).NE.0.)DFL(2)=-SIGN(TEMP2*XF(2))
      ONE=.5*(RPM(1)*(1.+GAMM)+RPM(2)*(1.-GAMM))
      OME=.5*(RPM(1)-RPM(2))*(1.-GAMM**2)/GAMM
      FCMASS*.5*(E+2.*XBAR*SRWT02)*(1047199*OME)**2
      G=EXING*OME*CMR*SRWT02*1.09623E-2
      CALL OUTCON
      IF(IQUIT.EQ.1)GO TO 160
      WRITE(0,165)ILOAD,IRR
      FORMAT(26H1 INPUT DATA FOR LOAN NO. 13,12H READING NO.,IT)
      165 FORMAT(26H1)
      WRITE(6,170)
      170 FORMAT(132H0 RPM OF RPM OF LOAN'S APPLIEN TO INNER
      INITIAL DISPLACEMENTS OF INNER WITH RESPECT TO OUTER ORBITAL
      2 ROTATIONAL/131H OUTER INNER ALONG Y ALONG Z
      3 ALONG X ALONG Y ALONG Z ABOUT X ABOUT Y VELOC
      4ITY VELOCITY/30X,9AMLQ LR IN RPM
      5 IN RADIAN RADIAN RPM
      WRITE(6,130)RPM(1),RPM(2),XF(2),XF(1),DFL11(2),DFL11(3),DFL11(1),
      1DFL11(4),DFL11(5),OME,OMR
      WRITE(6,180)GM,FC,FREE(2),FREE(3),FREE(1)
      180 FORMAT(155HU GYRO CENTRFUGAL 1. = DEFLECTION PERMITTED/
      15AH MOMENT FORCE ALONG X ALONG Y ALONG Z/20H
      2 LB*IN LB/1P5E12*4)
      KKK=0
      DO 190 K=1,3
      17(FREE(K).GT.0.)KKK=KKK+K**2
      190 CONTINUE
      IF(KKK.EQ.0.)GO TO 330

```

```

IF(FREE(1).EQ.0.)GO TO 330
SAV1=DFL(1)
SAV2=DFL(2)
SAV3=DFL(3)
SCL(1)=0.
IF((KKK.NE.1)GO TO 210
IF((FC*SATA/CBTA.LT.-XF(1))/XN)GO TO 320
WRITE(6,200)
200 FORMAT(82HU EXTERNAL THRUST IS NOT SUFFICIENT TO BALANCE INDUCED
1THRUST - PROBLEM ABANDONED)
GO TO 160
210 NOLOAD=0
DO 220 K=1,5
XF1(K)=0.
DO 220 L=1,5
DTV(L,K)=0.
DO 250 J=1,N
Y(J,J)
JPASS=J
P1=6.283185*(XJ-1.)/XN
SPH=SIN(PHI)
CPH=COS(PHI)
CALL ROLOU
IF((IQUITY)250,250,230
230 WRITE(6,240)ITX,J,(DFL(K),K=1,3)
FORMAT(19HU MAIN PROGRAM 240,216,1P3F12.4)
GO TO 160
250 CONTINUE
IF((KKK.GT.5)GO TO 260
E2=XF1(2)+XF(2)
COR2=E2/DIV(2,2)
DFL(2)=DFL(COR2)-COR2
IF((ABS(COR2)-TOL(4))310,200,290
260 IF((KKK.GT.10)GO TO 270
COR3=XF1(3)/DTV(3,3)
DFL(3)=DFL(COR3)-COR3
IF((ABS(COR3)-TOL(5))310,200,290
E2=XF1(2)+XF(2)

```

```

      FX=XF1(3)
      DTV=DTV(2,1)*DTV(3,3)-DTV(3,2)*DTV(2,1)
      C92=(E92*DTV(3,3)-ER3*DTV(2,3))/DET
      COR3=(DTV(2,2)*FR3-DTV(3,2)*FR2)/DET
      DFL(2)=DFL(2)-COR2
      DFL(3)=DFL(3)-COR3
      IF(ABS(C92)-TOL(4))280,290,290
      IF(ARS(COR3)-TOL(5))310,300,290
290  CONTINUE
      WRITE(6,300)KK,(DFL(K),K=1,3)
      300  FORMAT(19HU VAIN PROGRAM 30H,16,1P3F12.4)
      GO TO 160
      310  IF(XF1(1).LT.-XF(1))GO TO 320
      WRITE(6,270)
      GO TO 160
      320  DFL(1)=SAV1
      DFL(2)=SAV2
      DFL(3)=SAV3
      DO 530 IT(K=1,20
      NLOAD=0
      DO 340 K=1,5
      XF1(K)=0.
      DO 340 L=1,5
      DTV(L,K)=0.
      DO 370 J=1,N
      JPASS=J
      X'=J
      P,I=6.293185*(XJ-1.)/XIN
      SP4=SIN(PHI)
      CP4=COS(PHI)
      CALL ROLDRU
      IF(IQUIT)>70,350
      350  WRITE(6,360)ITFR,KKK,J,(DFL(K),K=1,3)
      360  FORMAT(19HU VAIN PROGRAM 36H,3T6,1P3F12.4)
      GO TO 160
      370  CONTINUE
      IF(KKK.EQ.1)GO TO 550
      IF(KKK.EQ.1)GO TO 380
      COR1=(XF1(1)+XF(1))/DTV(1,1)

```

```

DFL(1)=DFL(1)-COR1
IF(ABS(COR1)-TOL(3))550,530,530
IF(KKK.GT.4)60 TO 390
COR2=(XF1(2)+XF(2))/DTV(2,2)
DFL(2)=DFL(2)-COR2
IF(ABS(COR2)-TOL(4))550,530,530
IF(KKK.GT.5)60 TO 410
ER1=XF1(1)+XF(1)
ER2=XF1(2)+XF(2)
DET=DTV(1,1)*DTV(2,2)-DTV(2,1)*DTV(1,2)
COR1=(ER1*DTV(2,2)-ER2*DTV(1,2))/DET
COR2=(DTV(1,1)*ER2-DTV(2,1)*ER1)/DET
DFL(1)=DFL(1)-COR1
DFL(2)=DFL(2)-COR2
IF(ABS(COR1)-TOL(3))400,530,530
IF(ABS(COR2)-TOL(4))550,530,530
IF(KKK.GT.9)60 TO 420
COR3=XF1(3)/DTV(3,3)
DFL(3)=DFL(3)-COR3
IF(ABS(COR3)-TOL(5))550,530,530
IF(KKK.GT.10)60 TO 440
ER1=XF1(1)+XF(1)
ER2=XF1(2)+XF(1)
ER3=XF1(3)
DET=DTV(1,1)*DTV(3,3)-DTV(3,1)*DTV(1,3)
COR1=(ER1*DTV(3,3)-ER2*DTV(1,3))/DET
COR3=(DTV(1,1)*ER3-DTV(3,1)*ER1)/DET
IF(ABS(COR1)-TOL(3))430,530,530
IF(ABS(COR3)-TOL(5))550,530,530
IF(KKK.GT.13)60 TO 460
ER2=XF1(2)+XF(2)
ER3=XF1(3)
DET=DTV(2,2)*DTV(3,3)-DTV(3,2)*DTV(2,3)
COR2=(ER2*DTV(3,3)-ER3*DTV(2,3))/DET
COR3=(DTV(2,2)*ER3-DTV(3,2)*ER2)/DET
DFL(2)=DFL(2)-COR2
DFL(3)=DFL(3)-COR3
IF(ABS(COR2)-TOL(4))450,530,530
IF(ABS(COR3)-TOL(5))550,530,530
ERR(1)=XF1(1)+XF(1)

```

```

      F7R(2)=YF1(2)+XF(2)

      K0=3
      DO 470 K=1,3
      DO 470 L=1,3
      AA(L,K)=DTV(L,K)
      CALL SINWLT(AA,K0,ERR,CORR,IQUIT)
      IF(IQUIT)500,500,480
      480 WRITE(6,490)ITFR,KKK,(CORR(K),K=1,3),(DFL(K),K=1,3)
      490 FORMAT(19HU MAIN PROGRAM 490,216,1P4F12.4)
      GO TO 160

      500 DO 510 K=1,3
      510 DFL(K)=DFL(K)-CORR(K)
      DO 520 K=1,3
      IF(ABS(CORR(K))-TOL(K+2))520,530,530
      CONTINUE
      GO TO 550

      530 CONTINUE
      WRITE(6,540)KKK,(DFL(K),K=1,3)
      540 FORMAT(19H0 MAIN PROGRAM 540,16,1P3F12.4)
      550 CALL OUTPT
      GO TO 160
      E'ID

      SUBROUTINE ROLLOAD
      COMMON ALPHA,AA(5,5),A(5,5),ABIG,BFTA,R1,R2,RTA,BMTAU,RWTOP,
      1BT2AL,dGTMp,BR(2,60),C(2),CROWN,CBTa,CTAU02,CBMTAU,CWTOP2,
      2C9T2AL,CPH,CORR(5),CTH,COR(5),D,DRNP,DELD1(5),DFL1(5),CFL(3),
      3,7TV(5,5),DELTA,DEL(2)*DPDEL(2),D*DEL(2),D*DEL1(2),D*ELX,
      4D7TH,DA,DGX(2,60)*DELSAV(2,60),D*ID1(5),D*EL1(5),D*EL,DELLX,
      5E,EL(2),ERR(5),EFL,EX,ER(5),FLT,FLTn2,FREE(3),FC,
      6GAMM,GM,GWx, H1,H2,HERTZ(2,60),HR>,HN,IBR,ISTOP,IL0ND,IQUIT,
      7ICT,JPASS, KKK,N,NLOAD,OVR,Pn,PR(2),PRR,PX(2),
      AP(2,60),P3,PX3(60),P1OUT,P3OUT,R5,RHO,RY4,P1,R2,RP"(2)
      COMMON S3TA,S3AU02,SLANT,SBMTAU,SBTPAL,SAV1,SP4,STH,
      1 THD1(5),TAU02,TAU,TTAU02,TRT2AL,TNL(5),THETA,TH(2),TANTH,
      2T4SAV(2,60),THET,TMPBIG,V,UV,WIGHT,XN,XLT,YLE,XTAU,
      3XLTO2,XLE02,XMAR,XMASSI,XWASS2,XWASS,XINC1,XINC2,XINC3,XBAR1,
      4XRAR2,XRAR,XLEVER,XALPHA,XF(2),YF1(5),XW(2),XTH,YDEL,Y1,X2,
      5X"Y(2,60),XXX(2,2,60),X4SAV(2,60),X41OUT,X41G,Y43IG,Y4(2).

```

```

*Y'R
DOUBLE PRECISION B6TMP,C7H,DELT1,DEL,DPMFL,DMTH,GMX,HD
1,PX,P3,STH,TH,TANTH,VV,XM,XTH,XDEL
DOUBLE PRECISION A1D,A2D,A3D,A4D,BTAU,B"TAU,B"T2D,RAUTAU,BT2ALD,CRN,
ICTAUD,CRTAD,CRT2D,CRTUD,CRT2AD,DD,ELXY,DTTH,NSORT,DSN,DCOS,
2NY,DLUG,NETD,EN,FLD(2),EFLD,EXD,FLTO2D,FCO,GMD,PS1,PS2,P'DEL,P3TH
3,PS1DEL,PSITH,PS2DEL,PS2TH,P3DEL1,P3TH1,P3DEL2,P3TH2,PDEL,STAU2D,
4SATAD,SMT2D,SMTUD,SR12AD,SMINCD,SMD,TAU02D,TOLN(2),TEMPn,TEMP1D,
5XARD,XLEVND,XLE02D,XINC0,XHD,X1D,X2D,Y1n,Y2n
DIMENSION PXS(2),XYS(2),NPDELS(?),DMTHS(2),NPTHS(2),DMTHS(2)
IF((ICT.GT.U)GO TO 5
ICT=1
BTAD=BT1
BTM2D=BTM2D
BTMTUD=BTMTAU
BT2ALD=BT2AL
CNECROWN
CTAU2D=DCOS(BTAU02D)
CTADECRA
CMT2D=DCOS(BMT2D)
CRMUD=DCOS(BMTAU)
CRT2AD=DCOS(BT2ALD)
DND
EDE
ELD(1)=EFL(1)
ELD(2)=EFL(2)
FLTO2D=EFLT02
STAUD=DSIN(BTAU02D)
SNTAD=DSIN(BTAU)
SNTUD=DSIN(BMTAU)
SRT2AD=DSIN(BT2ALD)
TAU02D=TAU02
TOLD(1)=TOL(1)
TOLD(2)=TOL(2)
XARD=XARD
XLEVRD=XLEVER
XLE02D=XLE02
J=JPASS

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```

EFLD=11-120
XNCDF=FL7/30.00
XXA(1,2,3,4)=XX1
XXA(1,2,3,5)=XX2

```

20

```
    PYS(1)=P1OUT  
    X(1)=X1OUT  
    NLOAD=NLOAD+1
```

physiol

$$P(X(1)=P_1|O_{11})=x_{M1}^{(1)}(O_{11})$$

אלה (ארכיאו. כתה, פלט 1030, ירושלים, סת"ה, י"ד, א' 235, א' 236)

HEISST/HILF

$$XTH = TH(X)$$

סְבִּירָה

$$\text{dy}(x) = u \cdot du$$

$T^4(1) = \cdot 5D0 * TH$
 $T^4(2) = THETA - T$

$\text{DELT}(1) = \text{DELT}(2)$

176 לין (ב) מילר

卷之三

```

XHD=X1D+XINCD
SWINCD=1.00
BSTMPC=0.Dn
DO 110 L=1,31
  SWD=3.D0-SWINCD
  SWINCD=-SWINCD
  IF((L.EQ.1).OR.(L.EQ.31))SMD=1.00
  XHD=XHD-XINCD
  DELXD=XDEL+XHD*TANTH
  DTTHD=XHD/CTH**2
  IF(DABS(XHD).LE.FLT02D*CTH)GO TO 60
  TEMPD=XHD-HD*STH
  TEMP1DECERN**2-TEMPD**2
  IF(TEMP1D)30,30,50
30   IQUIT=1
      WRITE(6,40)IBR,ILOAD,IT,J,L,K,(NFL(W),W=1,3)
40   FORMAT(12H0 ROLOAD 40,6I6,1P3E12.4)
      RETURN
50   TEMP1D=NSORT(TEMP1D)
      DELXD=TEMP1D-HD*CTH+XDEL
      DTTHD=TEMPD*HD*CTH/TEMP1D+HD*STH
      IF(DELXn.LT.1.D-8)GO TO 110
      DXD=DD+2.D0*XHD*STA2D/CTAU2n
      EXD=ED+2.D0*XHD*SRTAD+DD*CBMT2D-DXD*CRTAD
      G'X=DXU*CRTAD/FXD
      IF(K.EQ.2)GMX=-DXD*CBMT1D/EXD
      TEMPD=3.D7*DELXD**1.11111/EFLD**.11111
      DC 70 ITF=1,20
      IF(TEMPn.LE.0.D0)GO TO 110
      A1D=ELD(K)*DXD*TEMPD*(1.n0+GVX)
      A1D=D3QRT(A1D)
      A2D=1.8864D0+DLG(EFLD*.5D0/A1D)
      A3D=ELD(K)*A2D*TEMPD
      A4D=(A3D-DELXn)/((A2D-.5D0)*ELD(K))
      TEMPD=TEMPD-A4D
      IF(DABS(A3D-DELXD)-TOLU(1)>0.70,70
      CONTINUE
      WRITE(6,80)IBR,ILOAD,IT,J,K,A4D,(DFL(W),W=1,3)
80   FORMAT(12H0 ROLOAD 80,5I6,1P4E12.4)

```

```

1000
      RETURN
      PY(K)=PX(K)+TEMPD*SMO
      XW(K)=XW(K)+XHD*TEMPD*SMO
      PDEL=SMO/((A2D-.5D)*FLR(K))
      NPDEL(K)=PDEL(K)+PCEL
      NPTH(K)=DPTH(K)+DNTHD*PDEL
      DNDL(K)=NMDL(K)+XHD*PDEL
      CPTH(K)=DNPTH(K)+XHD+DNHd*PDEL
      IF(TEMPD-QUTD)110,110,100
      QUTD=TEMPD-QUTD
      DLX(K,J)=ELX(J)
      HFRTZ(K,J)=.6366198D0*TEMPD/A1D
      Bn(K,J)=2.D0*A1D
      XSAV(K,J)=XHD
      CONTINUE
      IF(PX(K).EQ.0.0)GO TO 10
      TEMPD=XINC0/3.D0
      PX(K)=PX(K)+TEMPD
      XW(K)=XW(K)+TEMPD
      NPDEL(K)=PDEL(K)+TEMPD
      NPTH(K)=DPTH(K)+TEMPD
      DNDL(K)=NMDL(K)+TEMPD
      CPTH(K)=DNPTH(K)+TEMPD
      CONTINUE
      P3=(-XM(1)+XM(2))-GMD+FCD*XBOARD*CMT2D-(PY(1)-PX(2))*5D0*RD*ST.U2D
      1) XLEVRD
      P3THE(-(DPTH(1)+DPTH(2))*5D0*DPAUD-NWTH(1)-NWTH(2))/XLEVRD
      PS1DELE=DPODEL(1)*CBTAU-DPDEL(2)*VV*CRWTU0-FCD-P3*CBT2AD
      PS1THE=-DPTH(1)*SBTAD-DPTH(2)*CBVTU0-P3TH*SBT2AD
      PS2DELE=DPODEL(1)*SBTAD-DPDEL(2)*VV*SRWTU0+P3*CBT2AD
      PS2THE=-DPTH(1)*SBTAD-DPTH(2)*SBWTU0+P3TH*CBT2AD
      DFTD=P31DEL*PS2TH-PS2DEL*PS1TH/DET0
      Y1D=(PS1*PS2TH-PS2*PS1TH)/DET0
      Y2D=(PS1DEL*PS2-PS2DDEL*PS1)/DET0

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```

      DFL(1)=DEL(1)-Y1D
      DFL(2)=(DELT-A-DFL(1))*VV
      TH(1)=TH(1)-Y?
      TH(2)=IHETA-TH(1)
      IF(DARS(Y1D)-TOLD(1))130,140,14n
      IF(DARS(Y2D)-TOLD(2))160,140,14n
      CONTINUE
      130  WRITE(6,150) IPR,ILUAD,J,Y1D,Y2D,(DFL(m),m=1,3)
      140  FORMAT(13H0 ROLLOAD 150,316,1P5E12.4)
      141  IQUIT=1
      RETURN
      150
      150  17n  K=1*2
      P(K,J)=PX(K)
      X"4(K,J)=XM(K)
      DLSAV(K,J)=DFL(K)
      THSAV(K,J)=TH(K)
      PX3(J)=P3
      P3DEL1=(-MDEL(1)-DPDEL(1)*.5D0*DD*STAUPD)/XLEVRD
      P3TH1=(-D'TH(1)-DPTH(1)*.5D0*DD*STAU2D)/XLEVRD
      P3DEL2=(D'MEL(2)+DPDEL(2)*.5D0*DD*STAU2D)/XLEVRD
      P3TH2=(DMTH(2)+DPTH(2)*.5D0*DD*STAU2D)/XLEVRD
      A(1,1)=D'DEL(1)*CBTAD-P3DEL1*SRT2AD
      A(1,2)=D'PDEL(2)*CRMUD-P3DEL2*SRT2AD
      A(1,3)=D'P1H(1)*CRTAD-P3TH1*S9T2AD
      A(1,4)=DPTH(2)*CBMTUD-P3TH2*S9T2AD
      A(2,1)=D'DEL(1)*SBTAD+P3DEL1*CRT2AD
      A(2,2)=D'PDEL(2)*SBMTUD+P3DEL2*CRT2AD
      A(2,3)=DPTH(1)*SRTAD+P3TH1*CBT2AD
      A(2,4)=DPTH(2)*SBMTUD+P3TH2*CBT2AD
      A(3,1)=1.
      A(3,2)=1.*.7U/VV
      A(3,3)=0.
      A(3,4)=0.
      A(4,1)=0.
      A(4,2)=0.
      A(4,3)=1.
      A(4,4)=1.
      ER(1)=0.
      ER(2)=0.

```

```

      (3)=1.
      (4)=0.
    180 K=164
    180 L=164
    A(L,K)=A(L,K)
    I=4

190 CALL S,VULT(A,N7,ER,COR,TRUIT)
    I=(IQUAT)>20,220,200
    WRITE(6,210)IQR,ILCAD,J,(NFL(K),K=1,3)
200 FORMAT(13IU,110,116,1P3E12.4)
RETURN
220 GO TO(230,250),44
230 M=22
    D(240,K)=164
    D(240,L)=164
    A(L,K)=AA(L,K)
    DFLDELCOP(1)
    T'DELECR(5)
    FD(3)=U.
    FD(4)=L.
    G(1)=100
    DFLTHEOR(1)
    THTHECR(1)
    DFLD1(2)=C8TA+CPH
    DFLD1(3)=BTA+SPH
    DFLD1(4)=B*(E*S8TA+D*S8AU02)*S94
    DFLD1(5)=B*(F*S8TA+D*S8AU02)*CPH
    T(1)=14=S94
    THD1(5)=CPH
    255 K=1,2
    DPEELS(A)=DPDFL(K)
    DTHS(K)=HTH(K)
    DPEELS(A)=WDFL(K)
    DTHS(K)=LMTH(K)
    PYS(K)=PDX(K)
    XYS(K)=EXW(K)
    CNTINUE
    260 L=165

```

```

DP1D1(L)=(DPDELS(1)+DFLDL+DPTH5(1)*THDFL)*DEL01(L)+(NPDELS(1)*
1DFLTH+DPTH5(1)*THTH)*TH01(L)
260 DM1D1(L)=(DMDFLS(1)+DEL01+DMTHS(1)*THUFL)*DFLD1(L)+(DMDELS(1)*
1DFLTH+DMTHS(1)*THTH)*TH01(L)
270 XF1(1)=XF1(1)+PXS(1)*SBTA
XF1(2)=XF1(2)+PXS(1)*CBTA*CPH
XF1(3)=XF1(3)+PXS(1)*CBTA*SPH
TFYPE=.5*(E*SBTA+D*STA02)*PXS(1)+XYS(1)
XF1(4)=XF1(4)+TEMP*SPH
XF1(5)=XF1(5)+TEMP*CPH
IF(PXS(1).EQ.0.)RETURN
DO 280 L=1,5
DTV(1,L)=DTV(1,L)+DP1D1(L)*SRTA
DTV(2,L)=DTV(2,L)+DP1D1(L)*CRTA*CPH
DTV(3,L)=DTV(3,L)+DP1D1(L)*CBTA*SPH
TEMP=.5*(E*SBTA+D*STA02)*DP1D1(L)+DM1D1(L)
DTV(4,L)=DTV(4,L)+TEMP*SPH
DTV(5,L)=DTV(5,L)+TEMP*CPH
280 RETURN
END

```

SURROUNTING OUTCON

```

COMMON ALPHA,A(5,5),A(5,5),ABIG,BFTA,R1,R2,RTA,BMTAU,AMT02,
1BT2AL,BGTM,BB(2,60),C(2),CROWN,CBTA,CTAU02,CBMTAU,CMT02,
2CRT2AL,CPH,CORR(5),CTH,COR(5),D,DRnP,DELD1(5),DFL11(5),DFL(3),
3,DTV(5,5),DELTA,DEL(2),DMDEL(2),DPTH(2),DMTH(2),DLX,
4DDTH,DX,DLX(2,60),DELSAV(2,60),DL1(5),DM1D1(5),DELL,X,
5E,EL(2),ERR(5),EFL,EX,ER(5),FLT,FLT02,FREE(3),FC,
6GAMM,GM,6WY,H1,H2,HERTZ(2,60),HR7,HD,TBR,TSTOP,ILOAD,QUIT,
7ICT,JPASS,KKK,N,NLOAD,OME,OMR,PN,PR(2),PR,PX(2),
RP(2,60),P3,PX3(60),P1OUT,P3OUT,RS,PHO,PHW,R1,R2,RP"(2),
COMMON SRTA,STA02,SLANT,SBMTAU,S3T2AL,SV1,SAV?SAV3,SPH,STH,
1,THD1(5),TAU2,TAU,TTAU02,TRT2AL,TL(5),THFTA,TH(2),TANTH,
2TUSAV(2,60),THET,TMPBIG,V,VV,WFTGHT,XN,XLT,YL,XTAU,
3XLTO2,XLE02,XNAR,XMASS1,XMASS2,YMASS,XINC6,XINC6,YBAR1,
4XRAR2,XRAR,XLFEVER,XALPHA,XF(2),XF1(5),X"(2),XTH,XDEL,X?,
5XYVM(2,60),XXX(2,2,60),XHSAV(2,60),XMINUR,XX0(2),XH3IG,
QY,Q

```

DOUBLE PRECISION BGTM,CTH,DELTA,DEL,DANFL,DMDFL,DFL,DMTH,GUX,HD

1,DX,P3,STH,THETA,TH,TANTH,VV,XM,XTH,XEL

```

DOUBLE PRECISION C*N,CNTHD,DLD,FLTC,SNTHD,XRD,XRD
DFLR=UN5*0
THE T=0.
L 100 ITR=1,25
P1OUT=0.
PITH=0.
X'M1DEL=0.
X'1TH=0.
P1DEL=0.
PITH=0.
X'M1DEL=0.
X'1TH=0.
S'1TH=SIN(THET)
C'1TH=CUS(THET)
C*N=CROWN
CNTHD=CNTH
DLD=DELL
FLTD=FLTO2
S'NTHD=S'NTH
XLDEXL02
CALL XIREVE(CWN, CNTHD, FLTD, HD, KY, SNTHD, YN1, XN2, RD, XRD)
X1=XD1
X2=XD2
XX0(1)=X1
XX0(2)=X2
EFL=X1-X2
SWINC=1.
XINC=EFL/30.
XH=X1+XINC
T4PBIG=0.
DO 80 L=1,31
XH=XH-XINC
DY=D*D+2.*XH*TTAU02
EY=E*2.*XH*SRTA+D*CBMTO2-DX*C3T1
GAM2=DX*CRTA/EX
SM=3.-SWINC
SMINC=-SMINC
IF((L.EQ.1).OR.(L.EQ.31))SM=1.
DNTH=XH/CNTHD**2
IF(ABS(XH).LE.FLT02*CNTH)GO TO 40

```

```

TEMP=XH-RMH*SNTH
TF=MP1=CP0*N**2-TEMP**2
IF(TEMP1)10,1n,30
10  IQUIT=1
      WRITE(6,20)IBR,ILOAD,IIR,L'DELL,THET
20   FORMAT(12HU OUTCON 20,416,1P2E12.4)
      RETURN
30  TEMP1=ESORT(TEMP1)
      DFLX=TEMP1-RMH*CNTH+DELL
      DO THE TEMP+RMH*CNTH/TEMP1+RMH*SNTH
40   IF(DELX.LT.1.E-8)GO TO 80
      TEMP5=E7*JELLY**1.111111/EFL**.111111
      DO 50 ITG=1,20
      IF(TEMP.LF.=0.)GO TO 80
      A1=EL(1)*X*TEMP*(1.+GAM2)
      A1=ESORT(A1)
      A2=1.8864+ALOG(EFL*.5/A1)
      A3=EL(1)*A2*TEMP
      A4=(A3-DELX)/(A2-.5)*EL(1))
      TEMP=TEMP-A4
      IF((ABS(A3-JELX)-TOL(1))70,50,50
      CONTINU
50   WRITE(6,60)IBR,ILOAD,IIR,A4,DELL,THET
60   FORMAT(12HU OUTCON 60,316,1P3E12.4)
      IQUIT=1
      RETURN
70  P1OUT=P10UI+TF*MP*SM
      X'10UT=SYM10UT+XH*TEMP*SM
      P7EL=SM/(A2-.5)*EL(1)
      P1DEL=P1DEL+P7EL
      X'1DEL=SYM1DEL+XH*PDEL
      P1TH=P1TH+UDTH*PDEL
      X'1TH=SYM1TH+XH*DDTH*PDEL
      IF(TEMP.LF.=TEMP)GO TO 80
      TEMPBIG=TEMP
      A1G=2.*A1
      DFLX=DELL
      X'RIG=XH
      HnZ=TEMP/ADIG

```

40

CONTINUE

```

T=MP=X1INC/5.
P1OUT=P1011*TRMP
X'10UT=XW10UT*TRMP
P1DELE=P1011L*TRMP
X'10DEL=XM10EL*TRMP
P1THE=P1TH*TRMP
X'1THE=X'1TH*TRMP
PROUT=(-P1OUT*.5*D*STAJO+FC*XBAR*CWT02-F4-XM10UT)/XLEV0
PS1=P1OUT*CATA+FC-P3OUT*SBT2AL
PS2=P1OUT*SRTA+P3OUT*CRT2AL
PDEL=(.5*D*STA02*P10EL-XW10EL)/XLEV0
P1TH=(-.5*U*STA02*P1TH-XW1TH)/XLEVER
PS1DELE=CRTA*P1DEL-P3DEL*SBT2AL
P51TH=CRTA*P1TH-P3TH*SBT2AL
PS2DELE=SRTA*P1DEL+P3DEL*CRT2AL
PS2THE=-SRTA*P1TH+P3TH*CAT2AL
DET=PS1DEL*PS2TH-PS2DEL*PS1TH
COR1=(PS1*PS2TH-PS2*PS1TH)/DET
COR2=(PS10EL*PS2-PS2DEL*PS1)/DET
DFLELL-COR1
THEET=THFT-COR2
IE(LABS(COR1)-TOL(1))90,100,100
IF(LABS(COR2)-TOL(2))120,100,100
90 CONTINUF
100 WRITE(6,110)IPR,ILOAD,ITR,COR1,COR2,DFLL,THET
110 FORMAT(13HU OUTCON 110,316,1P4E12.4)
111 TQUIT=1
120 RETURN
END

```

ROUTINE OUTPUT

COMMON ALPHA,AA(5,5),A(5,5),ABIG,AFT1,P1,P2,ATA,BMTAU,AMTO2,
 1BT2AL,BCT2P,BA(2,60),C(2),CROWN,CBA,CTAU02,CMTAU,CMT02,
 2COT2AL,CPH,CORR(5),CTH,COR(5),
 3,RTV(5,5),DELTA,DEL(2),DDEL(2),DMDEL(2),DMTH(2),DELLX,
 4DMTH,DX,DLX(2,60),DELSAV(2,60),NP101(5),NW1D1(5),DELL,DELLX,
 5,F1(2),ERK(5),FFL,EX,E(5),
 6GMM,GM,GW,X, H1,H2,HERTZ(2,60),HR7,HD, IBL,ISTOP,ILoad,QUIT,
 7ITCT,JPASS, KKK,
 8ONE,OMR,
 9N,NOLOAD,

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8P(2,60),P1,PX3(60),P10UT,P30UT, RS,PHO,R4H,P1,R2,RP"(2)
COMMON SBT,A,STA02,SLANT,SBMTAU,SBT2AL,SAV1,SAV2,SAV3,SPU,STH,
1 THD1(5),TAU02,TAU,TTAU02,TTAU,TH(2),TANTH,
2THSAV(2,60),THE1,TMPBIG,V,VV,WFGHT,XN,XLT,YLE,XTAU,
3XLTO2,XLE02,XNAB,XMASS1,XMASS2,XMASS,XINC62,XINC61,YBARI,
4XRAR2,XRAR,XALPHA,XF(2),XF1(5),XW(2),XTH,XTEL,X1,X2,
5XW(2,60),XXX(2,2,60),XWSAV(2,60),XW10UT,XO(2),XH8IG,YV(2),
6Y'R

DOUBLE PRECISION RGTWP,CTH,DELTAD,DPDEL,DMDEL,DPTH,DMTH,GUX,HD
1,PX,P3,STH,TH,TANTH,VV,XM,XTH,XTEL
WRITE(0,10)ILOAD,IBR
FORMAT(0,0)OUTPUT DATA FOR LOAD NO.0,0,13,, BEARING NO.0,0,13,
10
WRITE(6,20)
20 FORMAT(0,0,15X,"REACTIONS OF BEARING ON SHAFT",29X,"TOTAL DISPLACE
1MENTS OF INNER WITH RESPECT TO OUTER",/
2ALONG Z ABOUT X ABOUT Y ALONG X ALONG Y ALONG
3Z ABOUT X ABOUT Y/6X,LR,INX,LR,10X,LB,AX,LB*IN,7X
4,LB*IN,9X,IN,10X,IN,10X,IN,AX,RADIAN5,5X,RADIANS")
DFL(1)=DFL(1)+DFL11(1)
DFL(2)=DFL(2)+DFL11(2)
DFL(3)=DFL(3)+DFL11(3)
WRITE(0,30)XF1(2),XF1(3),XF1(1),XF1(4),XF1(5),NFL(2),NFL(3),CFL(1
1)*DFL11(4)*DFL11(5)
30
FORMAT(1P11E12,4)
WRITE(6,40)
40 FORMAT(0,0)ROLL, ROLL, CONTACT LOAD
1ACT MOMENT, OUTER PATH EXTREMITY INNER PATH EXTREMITY CONT
2FLANGE/, NUMBER, AZIMUTH, OUTER, INNER, OUTER
3 INNER, X(1), X(2), X(1), X(2)
4 LOAD/17X,SEG,10X,LR,10X,LR,AX,LR*IN,7X,LB*IN,9X,IN
510X,IN,10X,IN,10X,IN,10X,IN,N
DO 70 J=1,N
XJ=J
PUT=360.*((XJ-1.)/YN
IF(P(2,J).EQ.0.)GO TO 70
WRITE(0,60)J,PHI,P(1,J),P(2,J),YMM(1,J),YMM(2,J),XXX(1,1,J),XXX(2
1,1,J),XXX(1,2,J),XXX(2,2,J),PX3(J)
70 FORMAT(17,5X,1P10F12.4)
CONTINUE

```


STHAR=JARC(STRU)

TEMP=TEMP111010.2r

**0

10 K=1

20 TURI
TEMP=TEMP(TEMP)

Y1=ETEP+H0*STHAR

Y2=-ETEP+H0*STHAR

YSTAR1=H0-USOPT(CP1*#2*YLF02n**2)

XSTAR1=XLF02D*CTH+JN1RBL*STHAR

YSTAR2=-XLE02n*CTH+DN3RL*STHAR

T=(X2n+1*XSTAR1)*3 TO 10

T,(X10*LE.FLTn2n*CTH) GO TO 10

T,(X10*GT.XSTAR1)*10*EXSTAR1

T,(X2n+1*L1*XSTAR2)*2*DSTAR2

T,(X2n+1*XSTAR2)*2*DSTAR2 AND. (X2n+1*R1*FLTn2n*CTH)) X2D=-XDEL*CTH/

11THAR

TYPE=AZ1N

Y1=2.12N

Y2=2.12N

R.TURI

END

END

END

END

END

END

END

END

SUBROUTINE SIMULT(AA,N,XX,XX,KK)

DIMENSION AA(5,5),B(5,5),XX(5),KL(5)

DOUBLE PRECISION A(5,5),B(5,5),XX(5),ROW(5),TEMP,AMPY

DO 10 I=1,N

R(I)=B(I,I)

DO 10 J=1,N

C(J,I)=AA(J,I)

DO 10 K=1,N

TEMP=0.0

DO 5051 K=1,N

TR(DARK1,J,K))-TFMP)

5051,5051,5052

TFMP=TRANS(A(J,K))

5052 CONTINUE

5053 K=1,N

A(J,K)=A(J,K)/TFMP

```

R(J)=R(J)/TEMP
5050 CONTINUE
5000 KNL(1)=1
      5001 DO 5002 IROW=2,N
      5002 KNL(IROW)=KOL(IROW-1)+1
      5004 DO 5025 KOUNT=1,N
      LARGST=N-KOUNT+1
      IERASE=KOL(1)
      JCOL=1
      5005 IF (N-KOUNT) 5035,5014,5006
      5006   AMPY =DABS (A(1,1))
      5007 DO 5010 IROW=2,LARGST
      5008 IF ( AMPY -DABS ( A(IROW,1))) 5009, 5010
      5009 JCOL=IROW
      AMPY =DABS ( A(IROW,1) )
      IERASE=KOL(IROW)
      5010 CONTINUE
      5011 IF (KOL(1)-IERASE) 5012,5014,5012
      5012 KNL (JCOL)=KOL(1)
      KOL(1)=IERASE
      5014 IF (A(JCOL,1)) 5015,5035,5015
      5015 AMPY=A(JCOL,1)
      5017 DO 5018 IROW=2,N
      ROW(IROW-1)=A(JCOL,IROW)/AMPY
      5018 A(JCOL,IROW-1)=A(1,IROW-1)
      ROW(N)=1.0/A*PY
      A(JCOL,N)=A(1,N)
      5019 DO 5022 IROW=2,N
      AMPY=A(IROW,1)
      5020 DO 5021 JCOL=2,N
      5021 A(IROW-1,JCOL-1)=A(IROW,JCOL)-AMPY*ROW(1:COL-1)
      5022 A(IROW-1,:)=-AMPY*ROW(N)
      5023 DO 5024 JCOL=1,N
      KNL (JCOL)=KOL (JCOL+1)
      5024 A(N,JCOL)=ROW(JCOL)
      5025 KOL(N)=IERASE
      5026 DO 5034 KOUNT=1,N
      5027 IF (KOL (KOUNT)-KOUNT) 5035,5036,5028
      5028 DO 5032 IROW=KOUNT,N

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5029 I= (KOL(IROW)-KOUNT) 5015,5030,5032
5030 J= 5015, COL=1,N
      IROW(1)=A(JCOL,IROW)
      A(JCOL,IROW)=A(JCOL,KOUNT)
5031 A(JCOL,KOUNT)=ROW(1)
      IERASE=KOL(KOUNT)
      KOL(KOUNT)=KOL(IROW)
      KOL(IROW)=IERASE
5032 CONTINUE
      TO 5034
5034 CONTINUE
      997 IF((KX-3)aa8,9000,998
      9000 KY=0
      RETURN
      998 ~0 5042 IROW=1,N
      5040 X(IROW)=0,J0
      5041 00 5042 JKOL=1,N
      5042 X(IROW) = X(IROW)+A(IROW,JCOL) * B(JCOL)
      KY=0
      RETURN
      ~0 6000 I=1,N
      5000 XY(I)=X(I)
      RETURN
      5035 KY=1
      RETURN
      END

```

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